

工學碩士 學位論文

# Hybrid GA

A Study on Optimal Facility Layout of Block Facility  
using Hybrid GA

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物流 工學科

李 容 煜

<b>1.</b>	.....	1
1.1	.....	1
1.2	.....	2
1.3	.....	4
<b>2.</b>	.....	5
2.1	.....	5
2.2	.....	7
<b>3.</b>	.....	11
3.1	.....	11
3.2	.....	13
3.3	(Genetic Algorithm) .....	15
3.3.1	.....	17
3.3.2	가 .....	18
3.3.3	.....	18
3.3.4	.....	23
3.4	(Simulated Annealing) .....	24
3.3.1	( T ) .....	26
3.3.2	(Perturbation) .....	26
3.3.3	.....	27
3.3.4	.....	27

3.5	Hybrid GA	.....	28
3.5.1	(GA)	.....	28
3.5.2	(SA)	.....	29
<b>4.</b>	<b>가</b>	.....	31
4.1	Parameter	.....	31
4.1.1	GA Parameter	.....	31
4.1.2	SA Parameter	.....	32
4.2		.....	33
<b>5.</b>		.....	44
		.....	46
		.....	49

## List of Tables

Table 1.1	.....	3
Table 4.1	Parameter .....	32
Table 4.2	5 10 , Hybrid GA .....	33
Table 4.3	15, 20, 30 , Hybrid GA .....	34

## List of Figures

Fig. 2.1	.....	6
Fig. 2.2	2가 .....	10
Fig. 3.1 Hybrid GA	.....	14
Fig. 3.2	.....	16
Fig. 3.3	.....	17
Fig. 3.4 (PMX)	.....	19
Fig. 3.5 (OX)	.....	20
Fig. 3.6 (CX)	.....	20
Fig. 3.7 Inversion	.....	21
Fig. 3.8 Insertion	.....	22
Fig. 3.9 Swap	.....	22
Fig. 3.10	.....	25
Fig. 3.11 Hybrid GA	Flowchart .....	30
Fig. 4.1 가 n=15 , Type 1 Hybrid GA GA	.....	35
Fig. 4.2 가 n=15 , Type 1 Hybrid GA SA	.....	35
Fig. 4.3 가 n=15 , Type 2 Hybrid GA GA	.....	36
Fig. 4.4 가 n=15 , Type 2 Hybrid GA SA	.....	36
Fig. 4.5 n=15 Type 1	.....	37
Fig. 4.6 n=15 Type 2	.....	37

Fig. 4.7	가 n=20	, Type 1	Hybrid GA	GA	38
Fig. 4.8	가 n=20	, Type 1	Hybrid GA	SA	38
Fig. 4.9	가 n=20	, Type 2	Hybrid GA	GA	39
Fig. 4.10	가 n=20	, Type 2	Hybrid GA	SA	39
Fig. 4.11	n=20	Type 1			40
Fig. 4.12	n=20	Type 2			40
Fig. 4.13	가 n=30	, Type 1	Hybrid GA	GA	41
Fig. 4.14	가 n=30	, Type 1	Hybrid GA	SA	41
Fig. 4.15	가 n=30	, Type 2	Hybrid GA	GA	42
Fig. 4.16	가 n=30	, Type 2	Hybrid GA	SA	42
Fig. 4.17	n=30	Type 1			43
Fig. 4.18	n=30	Type 2			43

# **A Study on Optimal Facility Layout of Block Facility using Hybrid GA**

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## **Abstract**

Facility layout is the early stage of system design that requires a mid-term or long-term plan. Since improper facility layout might incur substantial logistics cost including material handling and re-installment costs, due consideration must be given to decisions on facility layout.

Facility layout is concerned with how to arrange equipment necessary for production in a given space. Its objective is to minimize the sum of all the products of each equipment's amount of flow multiplied by distance. Facility layout also is related to the issue of NP-complete, i.e., calculated amounts exponentially increase with the increase of the number of equipment.

This study discusses Hybrid GA developed, as an algorithm for facility layout, to solve the above-mentioned problems. The Algorithm, which is designed to efficiently place equipment, automatically produces a horizontal passageway by the block, if a designer provides the width and length of the space to be handled. In

addition, this study demonstrates the validity of the Algorithm by comparing with existing algorithms that have been developed.

We present a hybrid GA approach to the facility layout problem that improves on existing work in terms of solution quality and method.

Experimental results show that the proposed algorithm is able to produce better solution quality and more practical layouts than the ones obtained by applying existing algorithms.



# 1

## 1.1

, 가 가

· ,

,

·

1955 , GNP 8% 가

20 50%가

·

10 30%

, 15 가 3

가 ,

,

·

가

가 가 가 가

Hybrid GA(Genetic Algorithm)

### 1.2

가 Muther (System Layout Planning)  
가

가  
(Quadratic Assignment Problem : QAP)

QAP

20 , 608 Job shop 가

가 가

(construction algorithm)

가

(improvement algorithm),

(hybrid algorithm), (graph theoretic algorithm) [29].

	ALDEP	Seehof and Evans 1967	
	CORELAP	Lee and Moore 1967	
	MAT	Edward et al. 1970	
	FATE	Block 1978	MAT FATE
	FLAT	Heragu and Kusiak 1986	3
	CRAFT	Armour and Buffar 1963	
	FLAT	Khalil 1973	
	MULTIPLE	Bozer et al. 1994	Spacefilling Curve
	FLAC	Scriabin and Vergin 1995	, FLAT
	BLOCKPLAN	Donaghey and Pire	2-opt

Table 1.1

Tate Smith가 1995 aspect ratio  
[28].  
1995 Suresh et al 가  
[22].  
QAP Crossover .  
1996 GA X GA  
CRAFT [12], 1997  
Bay [2].  
. .  
1997  
[1].

**1.3**

1 , .  
2 .  
3 .  
4 가 .  
, 5 .

2.1

(facilities layout)

, ,  
가 [13].

(facilities layout problem)

가 .

Fig. 2.1

가 . (facilities location)

(facilities design)

,  
가  
(facilities

system design)

(handling system design),

(layout design)

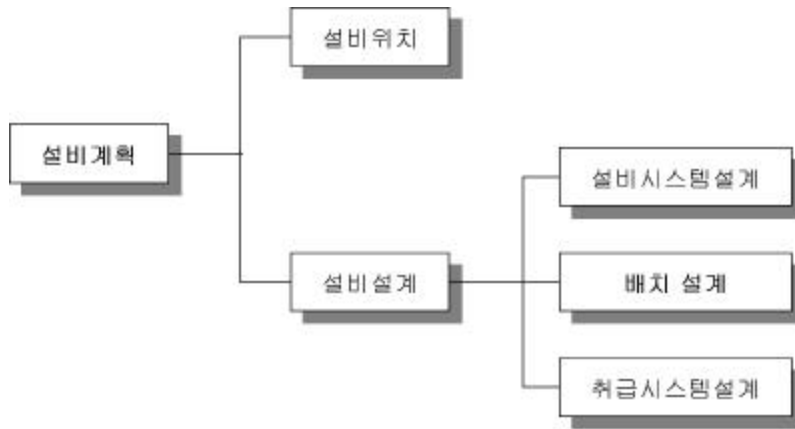


Fig. 2.1 (Tomkins et al. 1996)

가  
 가  
 가  
 가 NP-complete  
 가  
 가  
 가  
 (2.1)

$$\text{Min } F(x) = \sum_{i=1}^n \sum_{j=1}^n C_i \cdot a_{ij} \cdot d_{ij}, \quad i, j = 1, 2, 3, \dots, n \quad (2.1)$$

$$\begin{aligned}
 a_{ij} &= \text{ } i \text{ } j \\
 C_i &= \text{ } \\
 d_{ij} &= \text{ } i \text{ } j
 \end{aligned}$$

$$C_t = 1, a_{ij} = d_{ij}$$

( )

가

## 2.2

### 2.2.1

가 가

가

가

가

가

가

가

. Fig. 2.2

가

### 2.2.2

가 가

가 /

가

가

Hybrid GA

( ) ( Type 1 )

: 가 가

가 가

( )

가 가

가

가

가

가

가

( ) ( Type 2 )

: 가 /

가 /

가

가

가

가

가 가 /

가 /

가

가

가 /



가

가 /

가 /

가

가

가 /

가

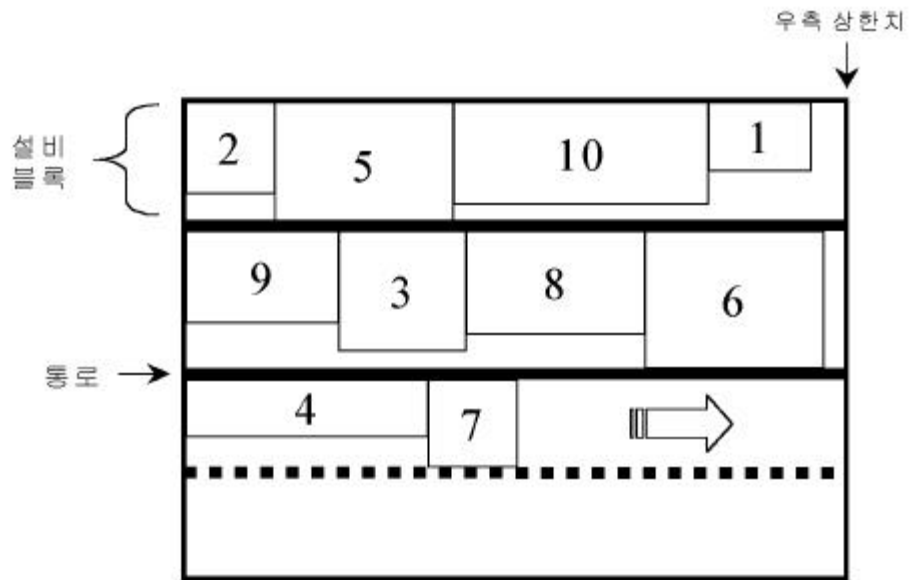
가

가

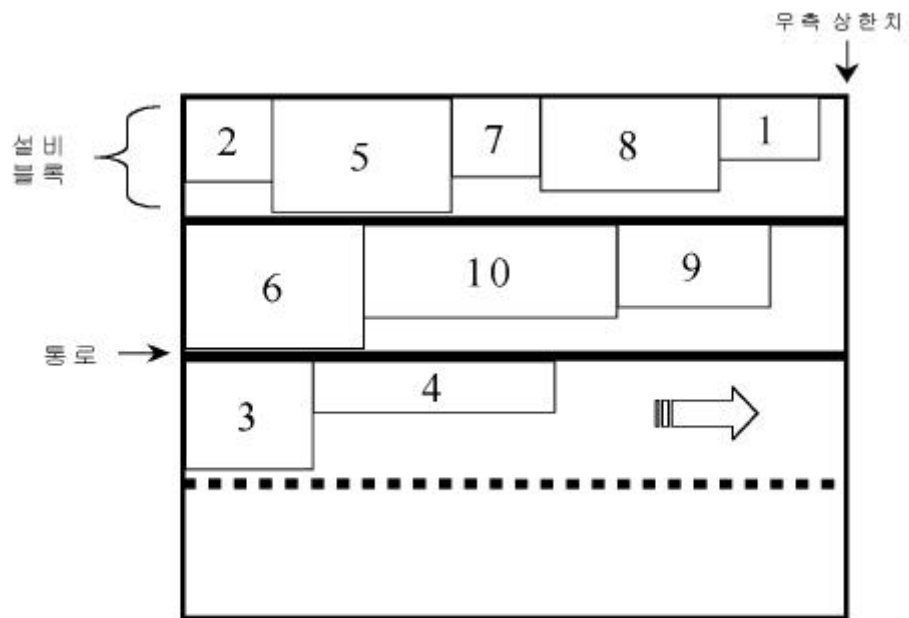
가

가

[1].



: Type 1 ( )



: Type 2 ( )

Fig. 2.2 2가

### 3

#### 3.1

가 .  
1950 1960  
1947 Dantzig  
1970 , 가  
TSP(Traveling Salesman Problem) 1970  
가 .  
NP-complete  
NP-complete  
가 ,  
가  
Genetic Algorithm, Simulated Annealing,  
가 15  
가 NP-Complete  
가 가 가

가 , 가 가

가 .

가 , 가

,

.

(Neural network)  
(neuron)

.

가 ,

가 가

(Genetic algorithm) 가

가 ,

가 .

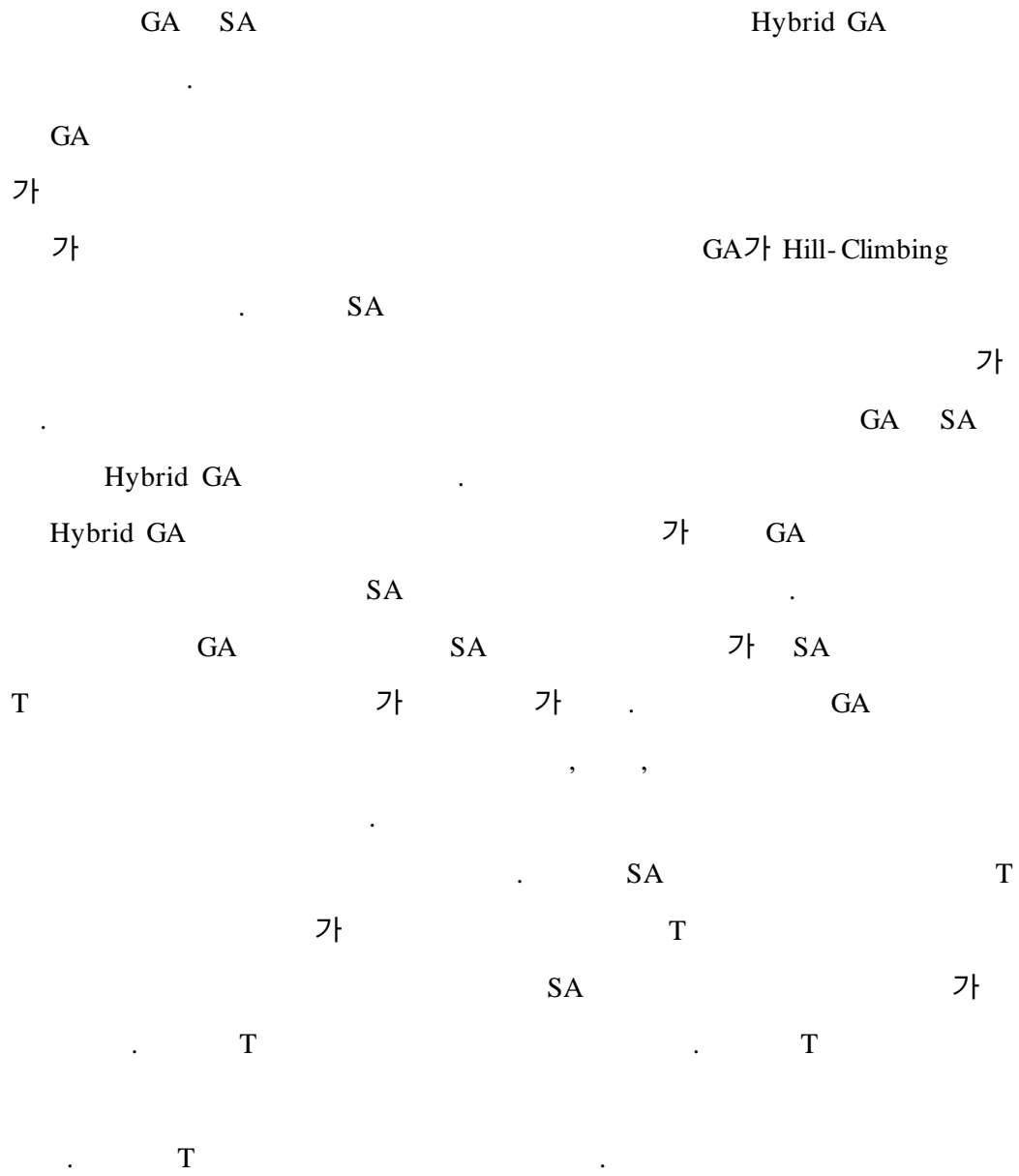
가 가

, , , ,

(Simulated annealing) (Iterative improve-  
ment)

가 .

가



**3.2**

- ( 3.2)
- 1 : , , , 가 .
  - 2 : .
  - 3 : 2 Genetic Algorithm , 가 .
  - 4 : 5 가 , 3 가 Genetic Algorithm .
  - 5 : Simulated Annealing .
  - 6 : , 5 가 Simulated Annealing .

Fig. 3.1 .

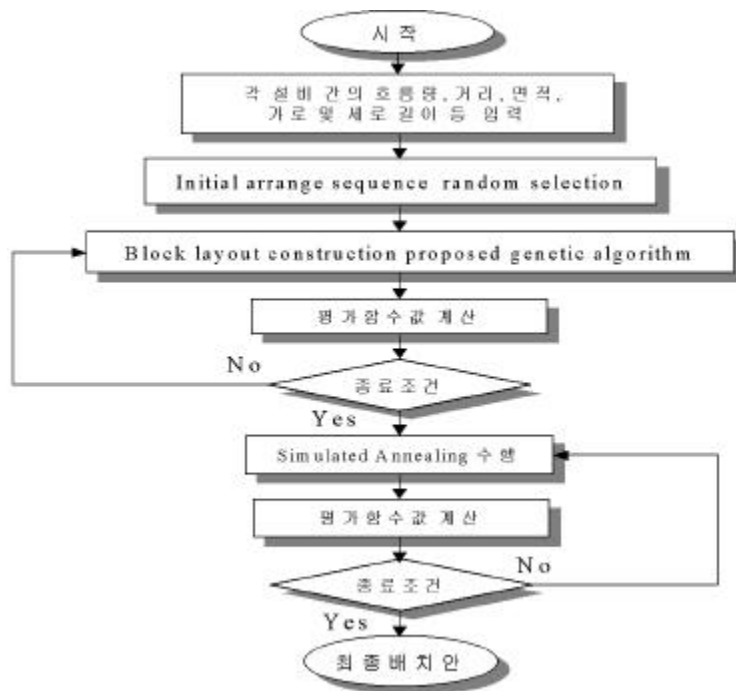


Fig. 3.1 Hybrid GA

### 3.3

### (Genetic Algorithm)

(Genetic Algorithm, GA)

. GA

(natural selection)

(survival of the fittest)

GA

가

. GA

(chromosome)

가

(reproduction),

(crossover),

(mutation)

가

가

GA가

, GA

(coding)

(point)

가

(fitness function)

. , GA

. , GA 가

(multimodal)

가

GA

GA

Fig. 3.2

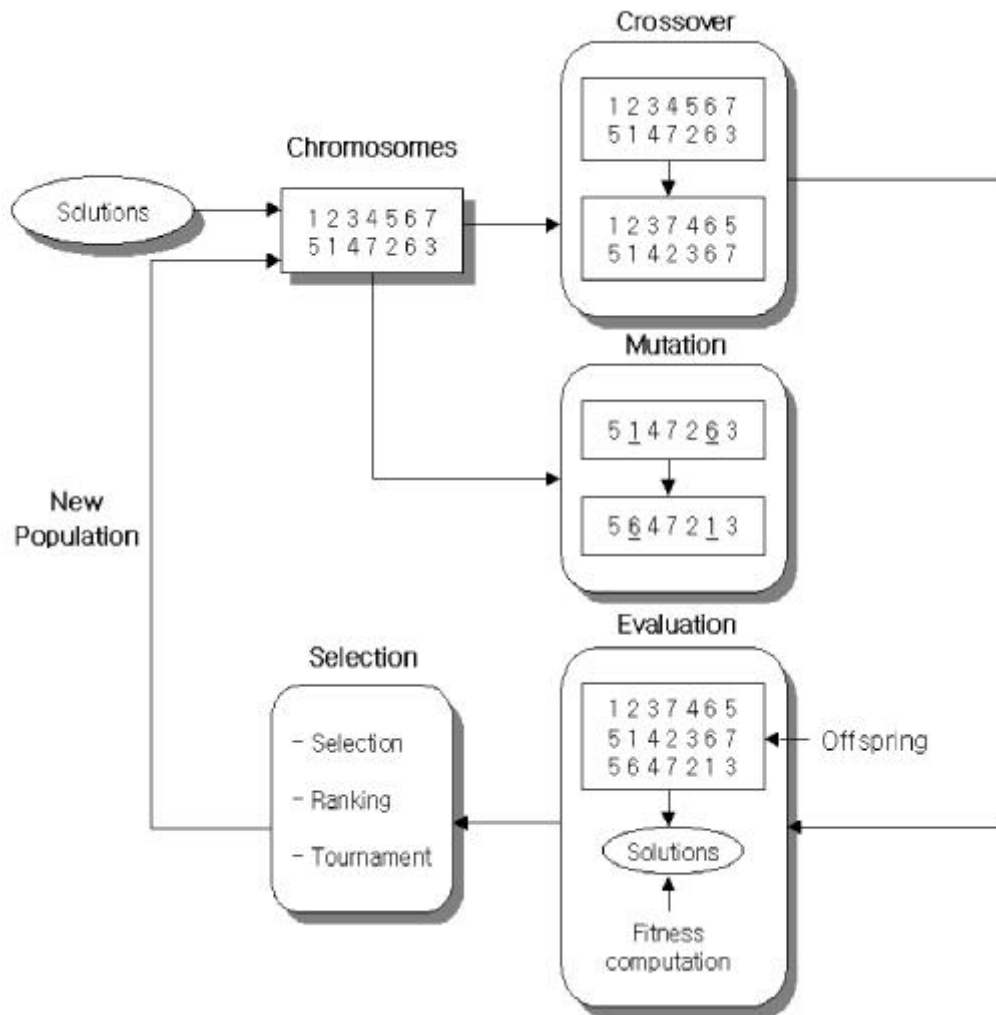
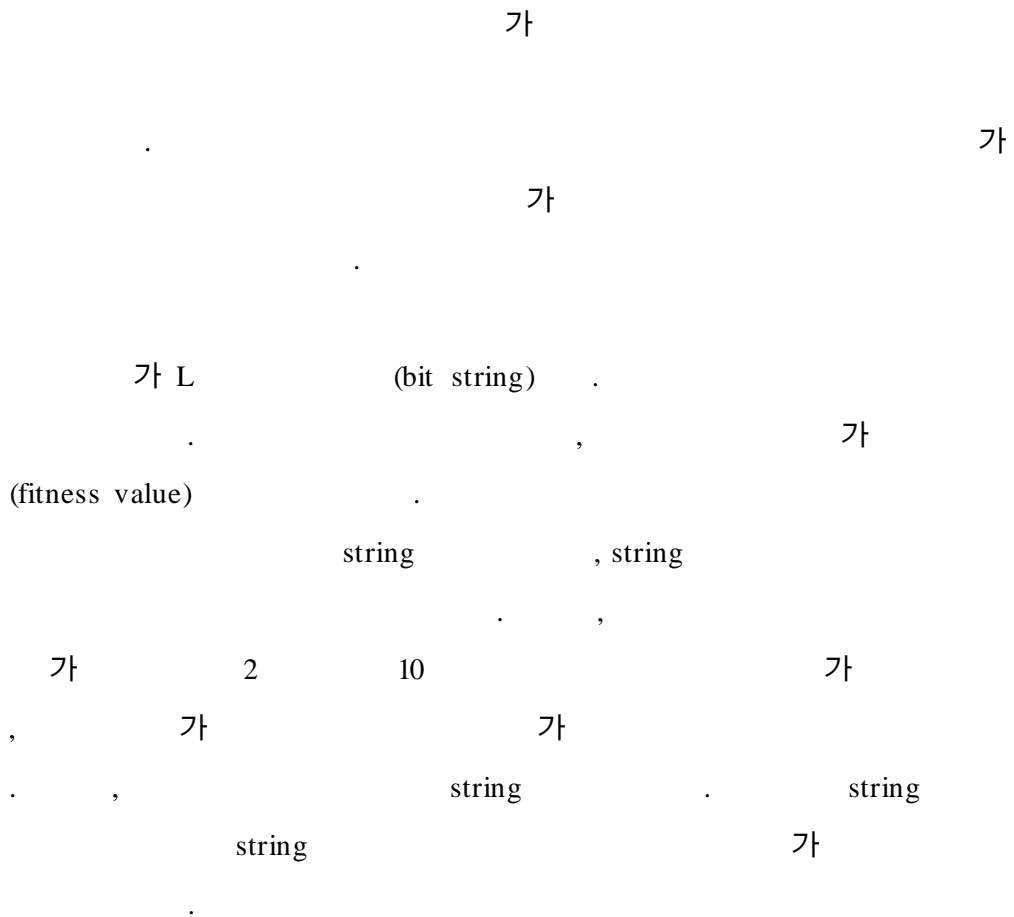


Fig. 3.2



### 3.3.1



solutionstring → 

1	3	2	4	10	5	9	6	7	8
---	---	---	---	----	---	---	---	---	---

Figure 3.3

### 3.3.2 가

가

$$\text{Min } F(x) = \sum_{i=1}^n \sum_{j=1}^n C_i \cdot a_{ij} \cdot d_{ij}, \quad i, j = 1, 2, 3, \dots, n$$

$$a_{ij} = \quad i \quad j$$

$$C_i =$$

$$d_{ij} = \quad i \quad j$$

$$C_i = 1, \quad a_{ij} \quad d_{ij}$$

( )

가

### 3.3.3

(crossover),

(mutation)가

(selection),

(1) (crossover)

(crossover)

가

가

2

(character preservingness)

(crossover neighborhood)

(PMX : Partially matched crossover)

TSP

A B

2

(cut - point)

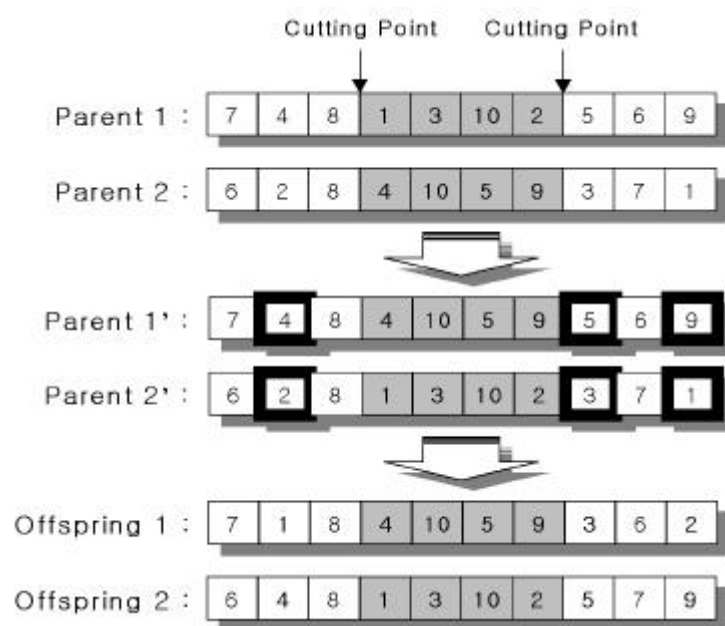


Fig. 34

(PMX)

(OX : ordered crossover)

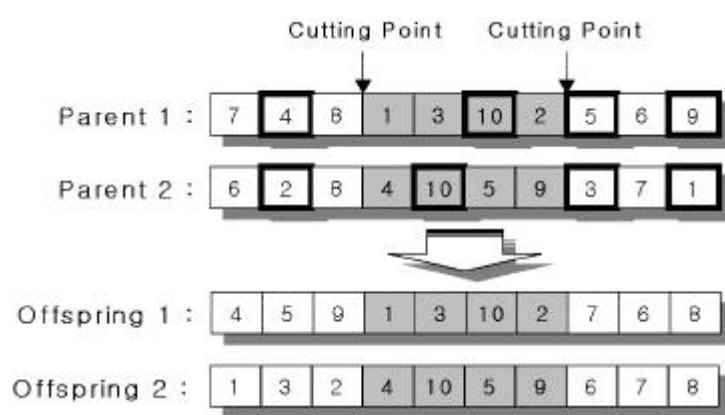


Fig. 3.5 (OX)

(CX : cycle crossover)

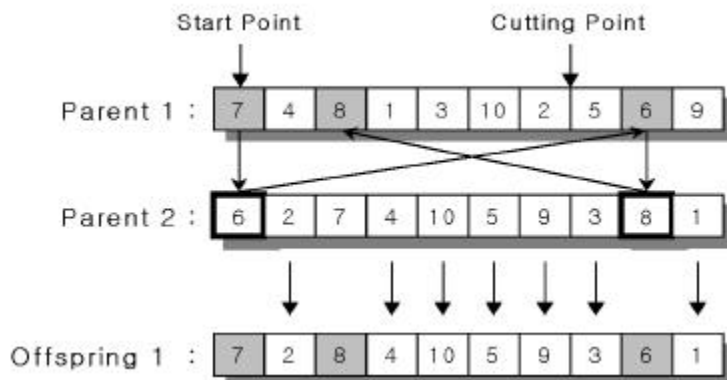


Fig. 3.6 (CX)

(2) (mutation)

가

가

가

가

Inversion, Insertion,

Swap

Inversion

(cutting point)

가

가

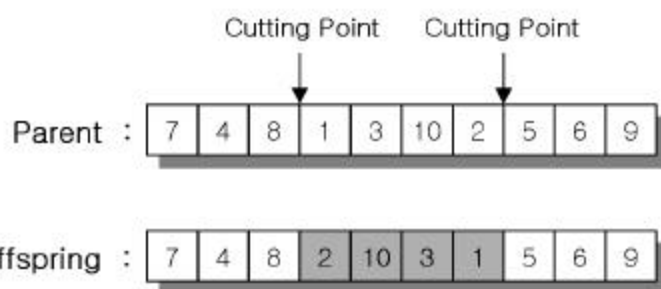


Fig. 3.7 Inversion

Insertion

string

가

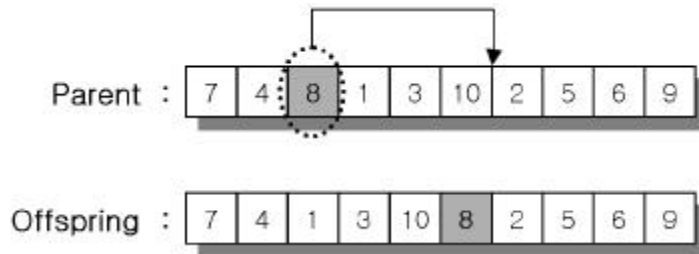


Fig. 3.8 Insertion

Swap

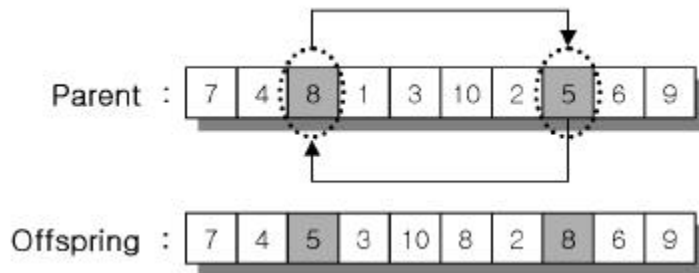


Fig. 3.9 Swap

(3) (selection)

(natural selection)

(ranking selection)

(tournament)



### 3.4

### (Simulated Annealing)

(Simulated Annealing, SA)

가 .  
가 .  
가 .  
가 .  
T 가 .  
가 .  
가 .  
가 .  
가 .  
SA 가 .  
(Annealing Schedule)  
SA Hill-Climbing Steepest Descent . ,  
SA .  
Metropolis .  
Metropolis .



Hill Climbing Steepest Descent

가

Metropolis

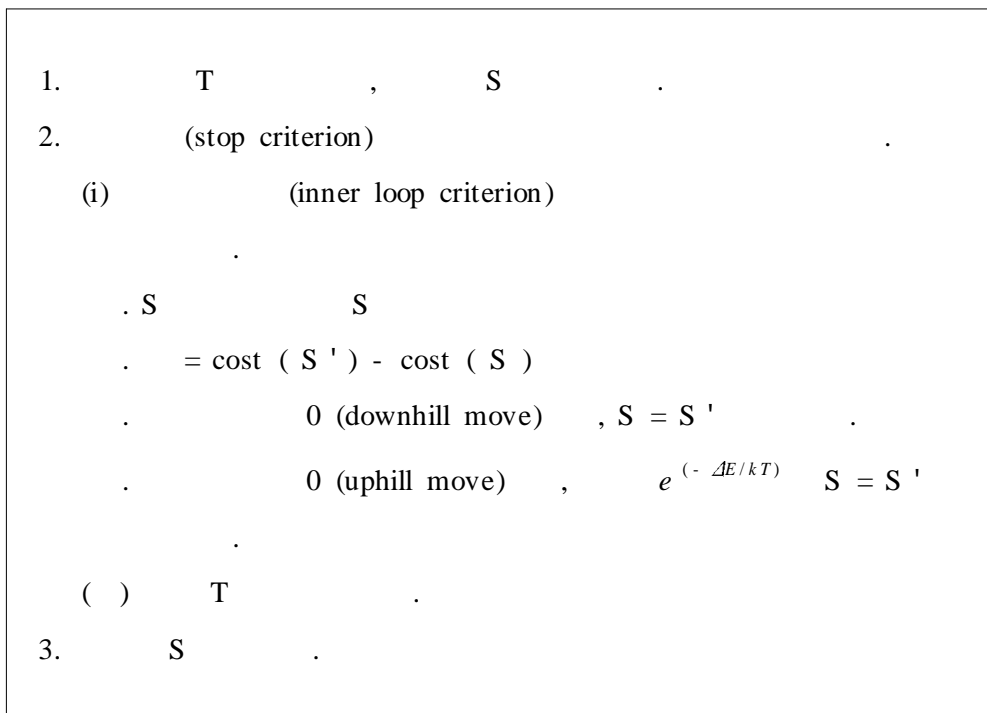


Fig. 3.10

SA

가

가

Metropolis

Metropolis

SA

가 ,

,  
,

가

Hybrid GA

GA

가

string

가 SA

### 3.4.1 (T)

Hybrid GA

T

$$\exp(-\Delta E/kT) \cong 1$$

가

가

(T) 5

### 3.4.2 (Perturbation)

Fig. 3.10

2( )

S'

가

가  
 (mutation)  
 string .

### 3.4.3

.L , L  
 가 L  
 가 .

Fig. 3.10 2 ( ) (3.1)

$$T' = r \cdot T \quad (3.1)$$

$T'$  ,  $r$  0.95 0.99 .

### 3.4.4

Fig. 3.10 2

, 가  
 , ( ) ,

가

T

가

### 3.5 Hybrid GA

#### 3.5.1 (Genetic Algorithm)

- [ 0] . , , 가 / , , population , 가 , .
- [ 1] Population string .
- [ 2] string (crossover) .
- [ 3] (mutation) .
- [ 4] 가 .
- [ 5] Reproduction Rule .
- [ 6] [ 7] , , [ 3] 가 .
- [ 7]

### 3.5.2

### (Simulated Annealing)

[ 1] GA 가 , T  
L .

[ 2] .

[ 3] (inner loop criterion)

[ 3-1] GA 가 .

[ 3-2] GA 가 .

[ 3-3] 가 0 ,  
 ,  $e^{(-\Delta/kT)}$  .

[ 4] T .

[ 5] [ 6] ,

, [ 3] 가 .

[ 6]

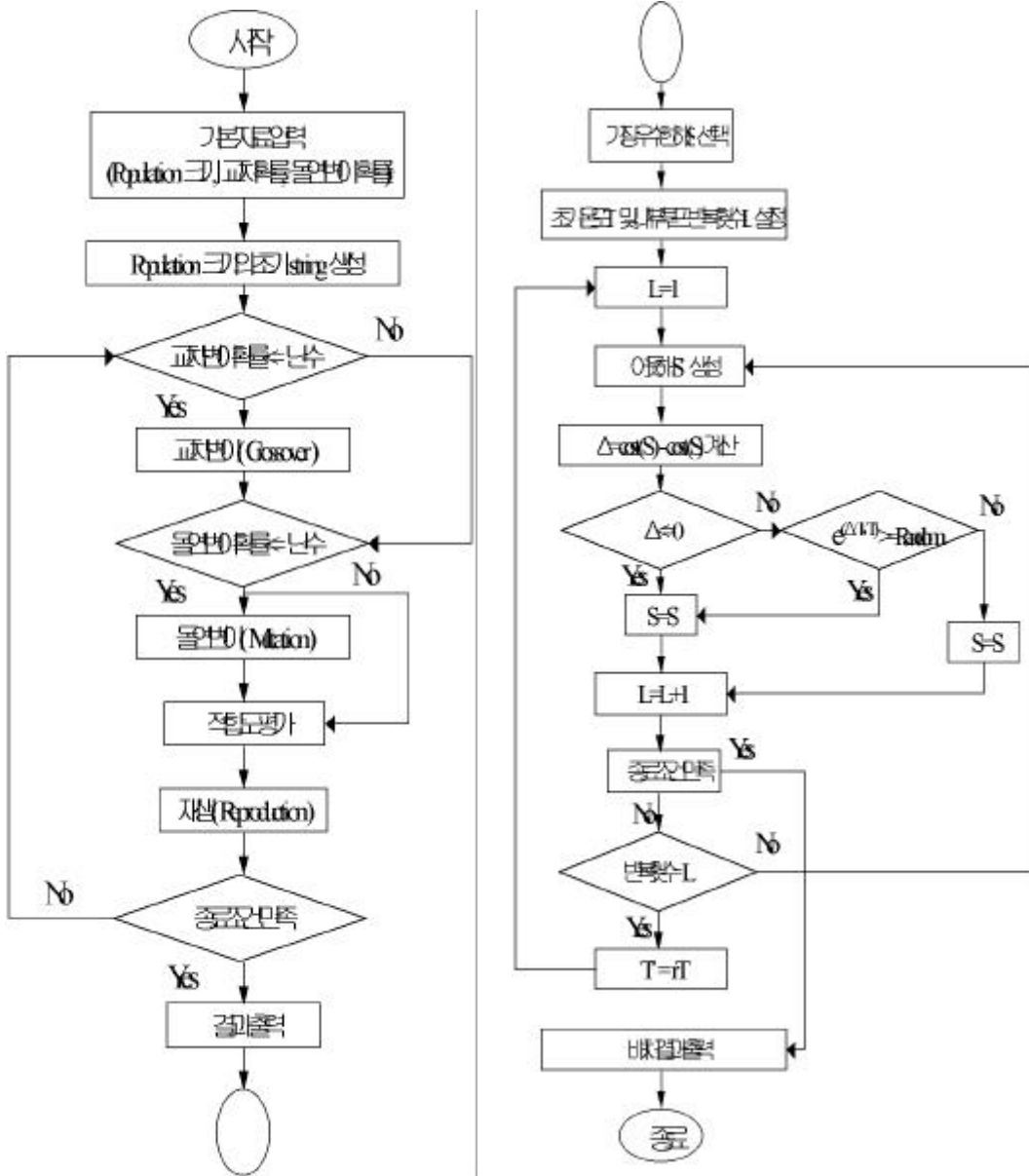


Fig. 3.11 Hybrid GA

Flowchart

## 4 가

Hybrid GA

가 5, 6, 7, 8, 9, 10, 15, 20, 30

Tam GA

(

[1][15][21]).

Hybrid GA Visual C++ , 450Mhz, RAM  
128 Mbyte, Windows 98 가 PC .

### 4.1 Parameter

#### 4.1.1 Genetic Algorithm Parameter

parameter .

Population size 10 100 10 가 50  
50 50  
50 Population size 50  
Crossover rate Mutation rate 0.0 1.0 0.05 가  
Crossover rate 0.2 0.3 , Mutation rate 0.1  
가 parameter  
Population size 50 가  $n = 20$  Crossover  
Rate = 0.1 0.3, Mutation Rate = 0.1 Crossover Rate 0.01  
, Population size = 50, Crossover Rate = 0.25, Mutation Rate = 0.1  
가 generation 5000

, 가 2000 가 .

#### 4.1.2 Simulated Annealing Parameter

가 .

20 (T) 5

500 . (r) 0.9, 0.5

.

,

, 가

가 . ,

가 , (Boltzmann factor)

$$\exp\left(-\frac{\Delta E}{K_B \cdot T}\right)$$

. (ΔE)가 ,

$K_B$  가 .

(n)			( $K_B$ )	
15	5	0.9	80 90	500
20	5	0.9	150 160	500
30	5	0.9	300	500

Table 4.1 Parameters



4.2

가 가 5 10

Table 4.2

가 5 10

가

가

(n)			GA (Woo and Park)	Hybrid GA	String	
5	Type 1	512	512	512	1-2-3-4-5	100%
	Type 2	533	533	533	2-1-3-5-4	100%
6	Type 1	896	896	896	1-2-3-4-5-6	100%
	Type 2	884	884	884	3-2-5-1-4-6	100%
7	Type 1	1319	1319	1319	1-4-5-2-3-7-6	100%
	Type 2	1230	1230	1230	1-4-5-2-3-7-6	100%
8	Type 1	<b>1940</b>	<b>1965</b>	<b>1940</b>	1-2-5-8-7-4-6-3	100%
	Type 2	1806	1806	1806	5-1-2-6-4-8-7-3	100%
9	Type 1	2673	2673	2673	1-2-5-8-7-3-6-4-9	100%
	Type 2	2664	2664	2664	1-2-5-8-7-3-6-9-4	100%
10	Type 1	3709	3709	3709	1-2-5-8-7-3-6-10-9-4	100%
	Type 2	<b>3519</b>	<b>3591</b>	<b>3519</b>	10-9-1-8-7-3-4-6-2-5	100%

Table 4.2 가 5 10 , Hybrid GA

PC 15 30

Table 4.3

, Tam 17 35% 가 , Woo and  
 Park GA 2 9% 가 .  
 , 20  
 , 가  
 가 Hill-Climbing

SA GA

가

(n)		Tam	Woo & Park GA	Hybird GA		Efficiency of Hybrid GA	
						Tam	Woo & Park
15	Type 1	13762	9120	8847	8911	35.71%	2.99%
	Type 2	12240	9855	9521	9582	22.21%	3.39%
20	Type 1	26921	21885	20029	20696	25.60%	8.48%
	Type 2	28646	22656	21698	21926	24.25%	4.23%
30	Type 1	55668	50492	46286	46545	18.06%	9.66%
	Type 2	58824	52884	48814	49324	17.02%	7.70%

Table 4.3 가 15, 20, 30 , Hybrid GA

Hybrid GA

Type 1, Type 2

$n = 15, 20, 30$  , GA

SA

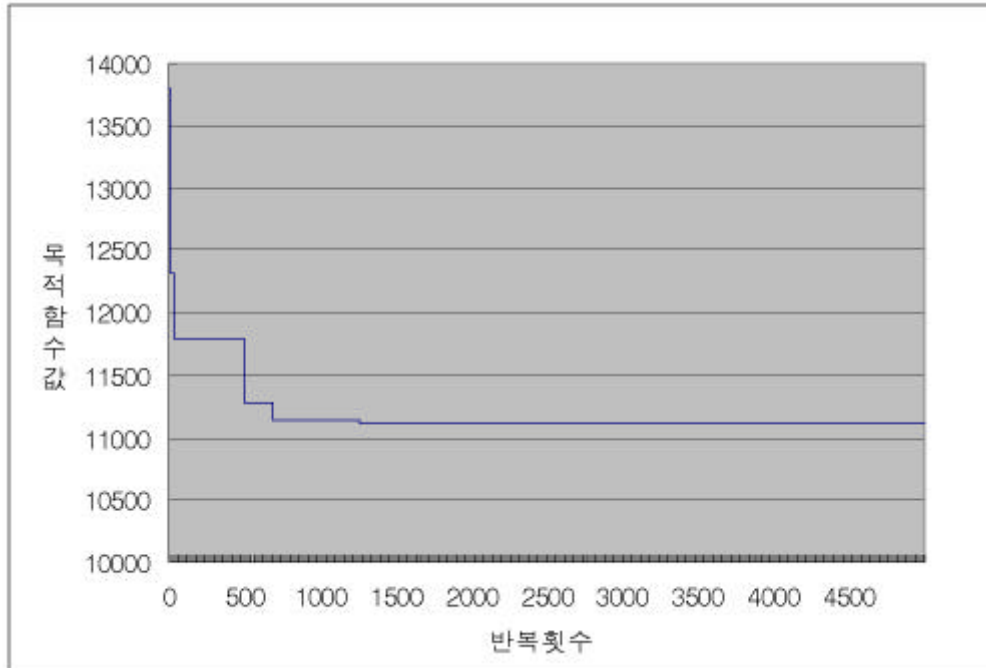


Fig. 4.1 n=15, Type 1 Hybrid GA GA

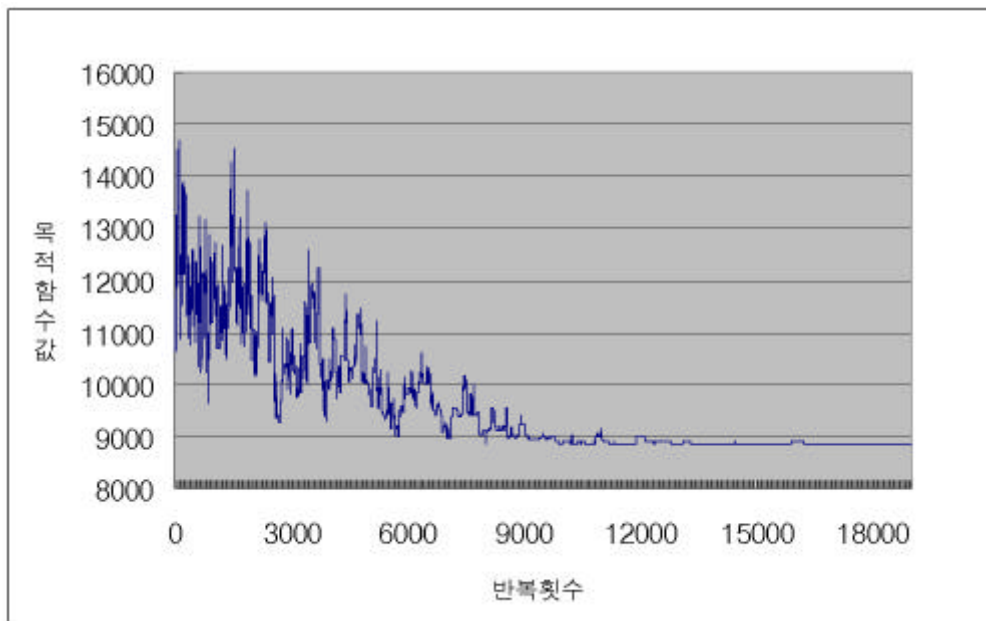


Fig. 4.2 n=15, Type 1 Hybrid GA SA

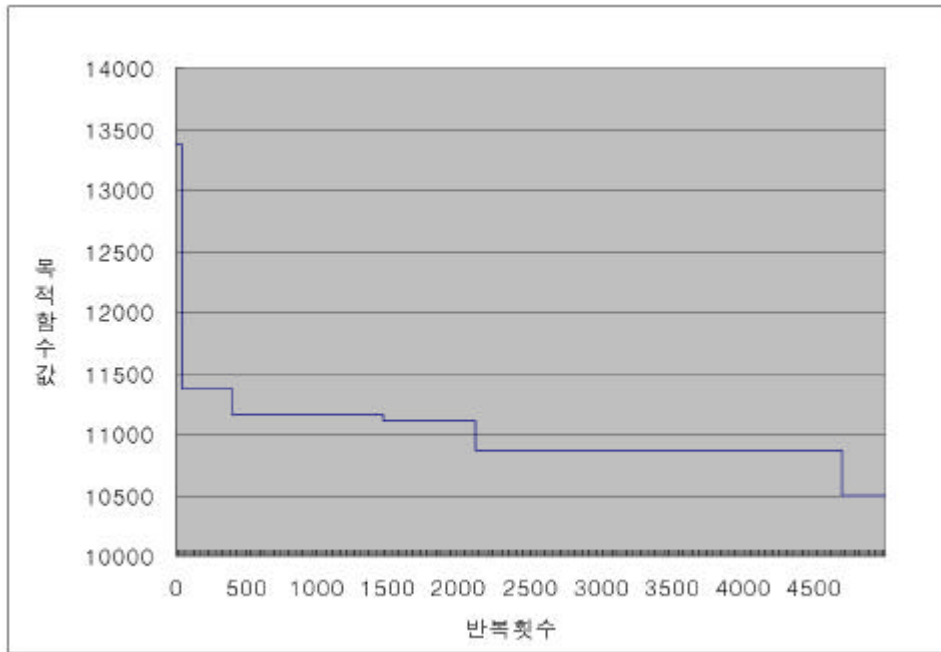


Fig. 4.3 n=15 , Type 2 Hybrid GA GA

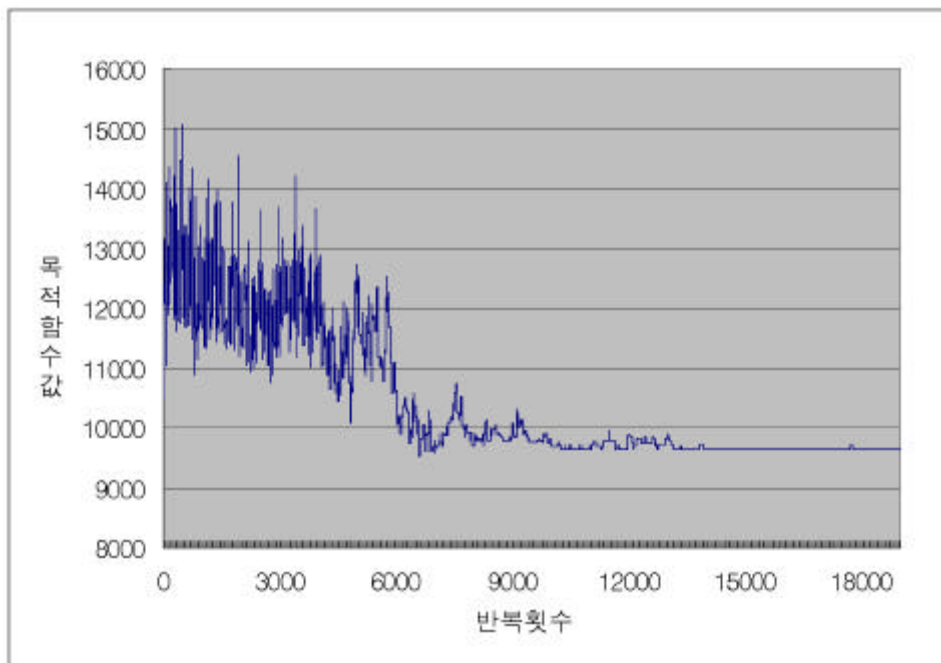


Fig. 4.4 n=15 , Type 2 Hybrid GA GA

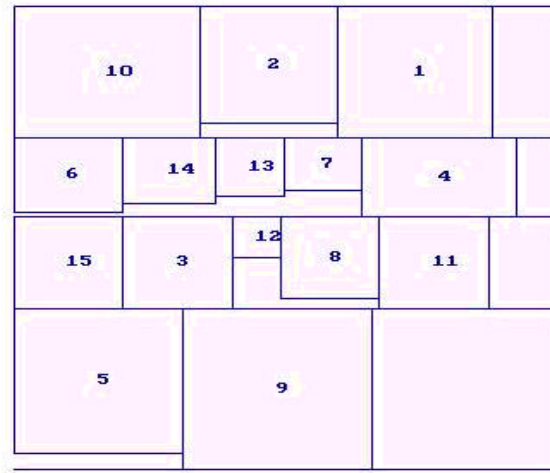


Fig. 4.5      n=15      Type 1

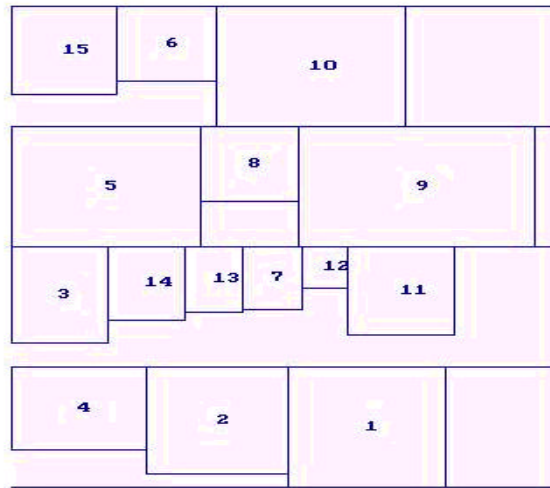


Fig. 4.6      n=15      Type 2

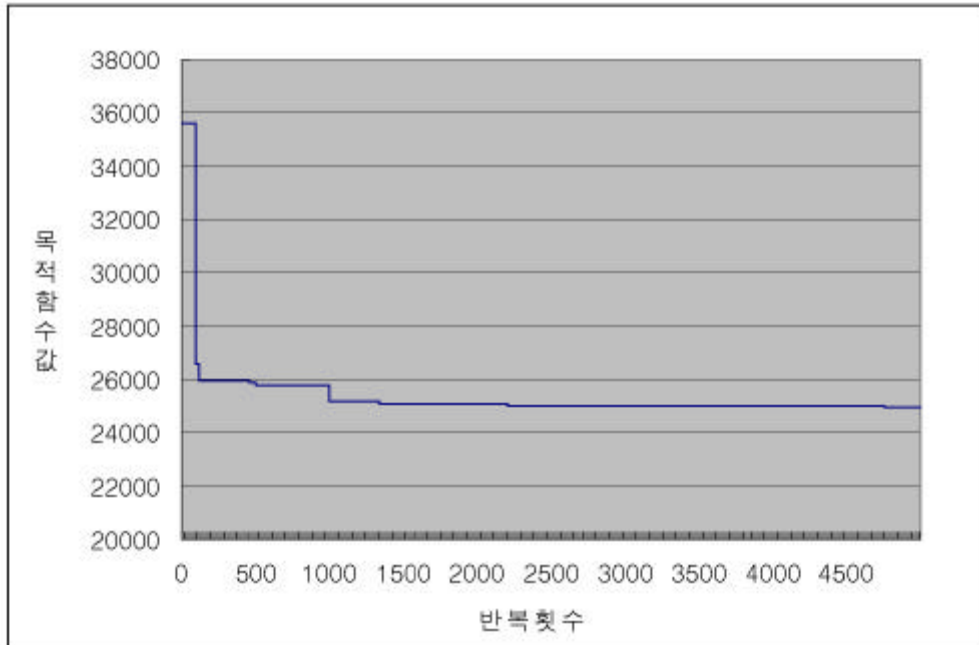


Fig. 4.7  $n=20$ , Type 1 Hybrid GA GA

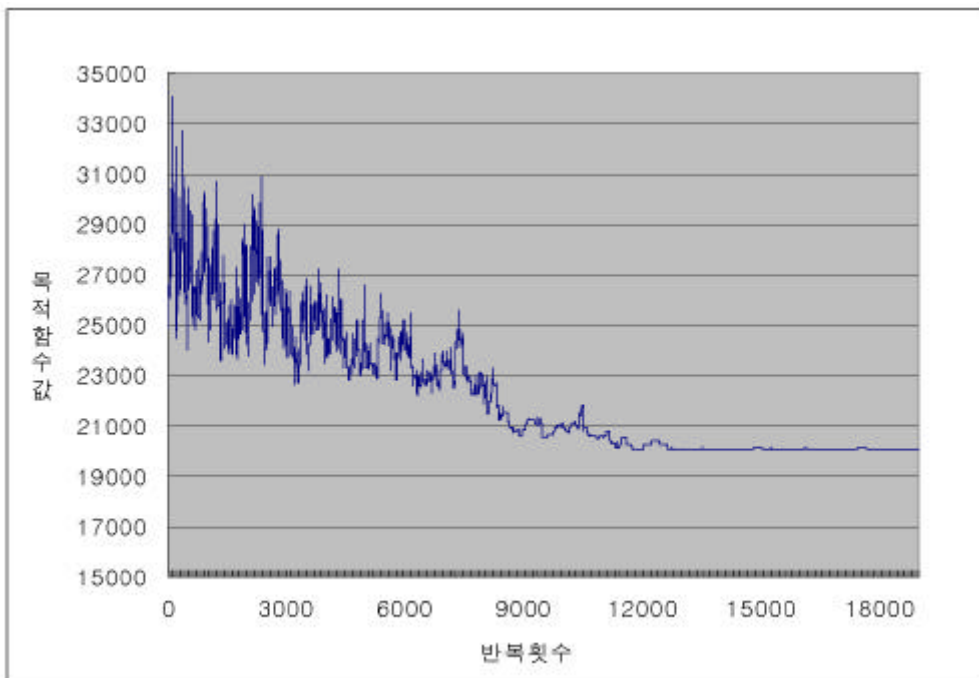


Fig. 4.8  $n=20$ , Type 1 Hybrid GA SA

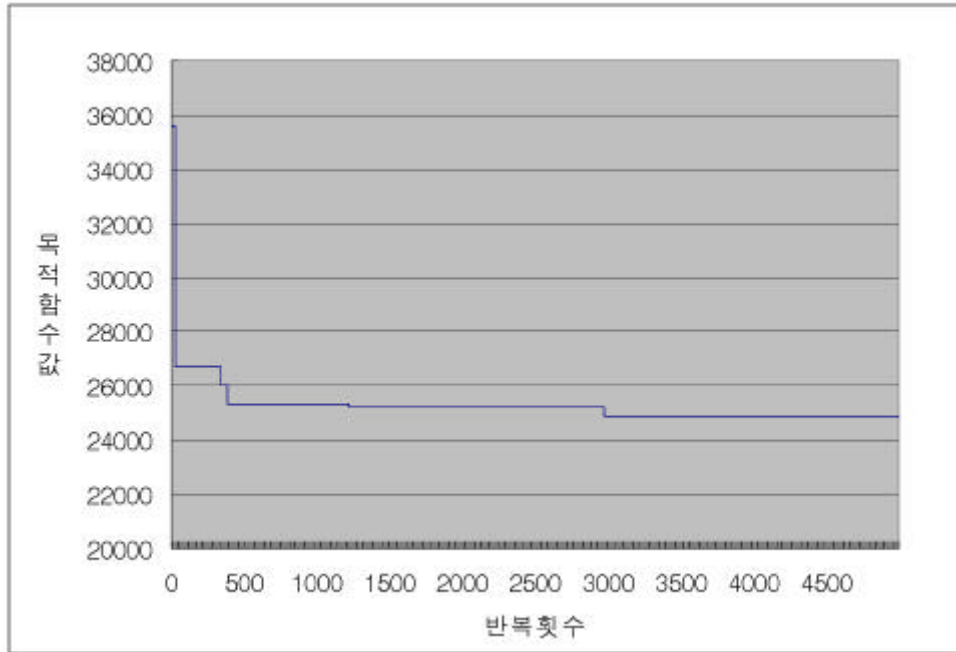


Fig. 4.9  $n=20$ , Type 2 Hybrid GA GA

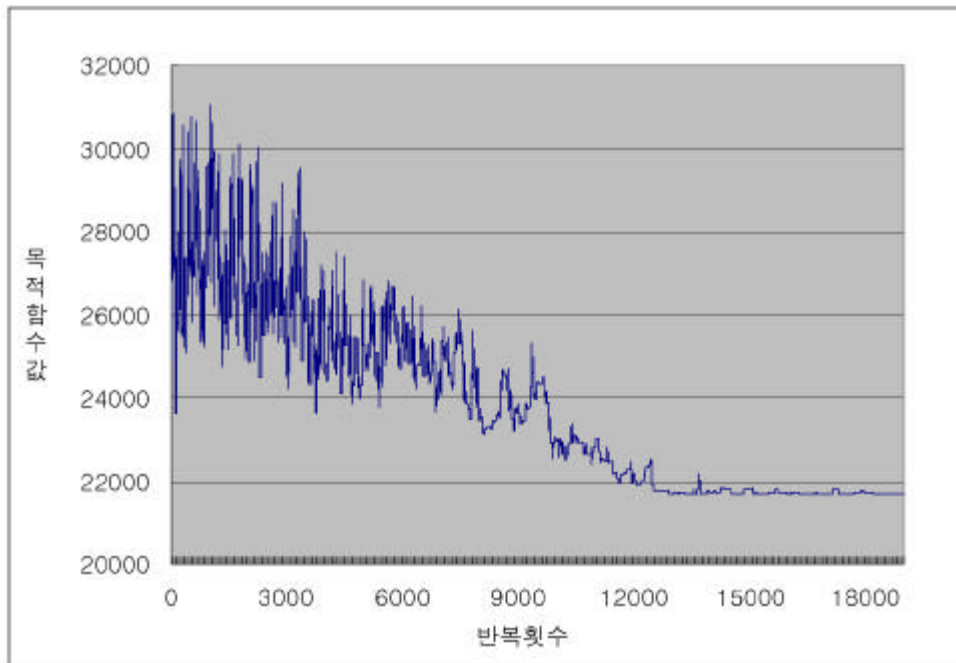


Fig. 4.10  $n=20$ , Type 2 Hybrid GA SA

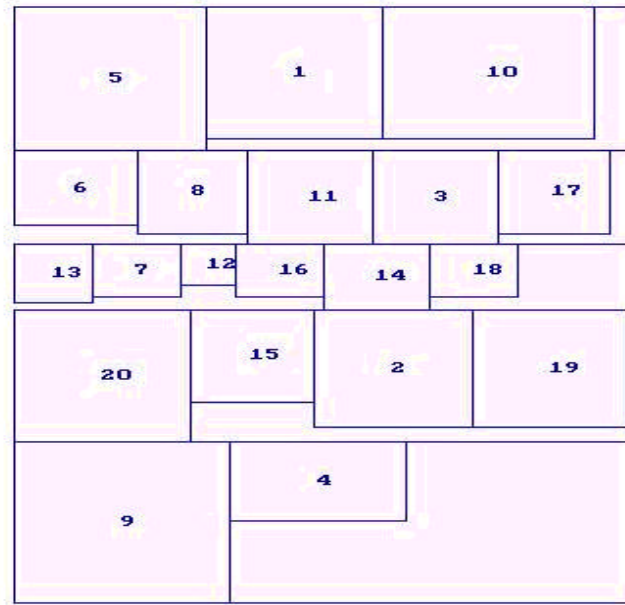


Fig. 4.11      n=20      Type 1

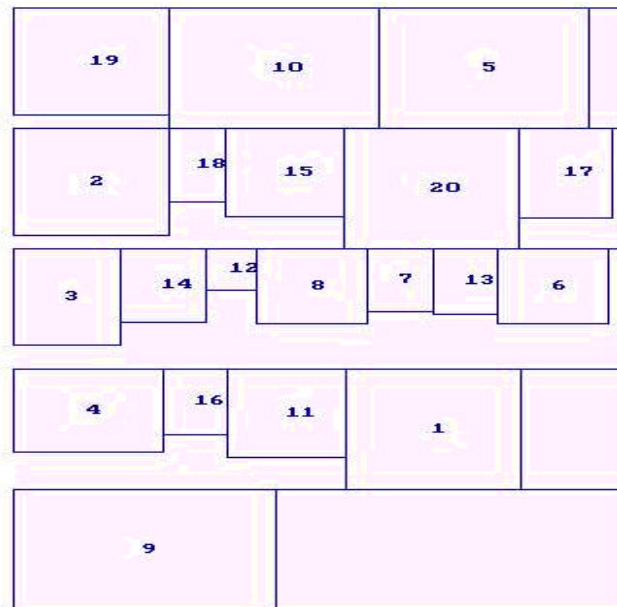


Fig. 4.12      n=20      Type 2



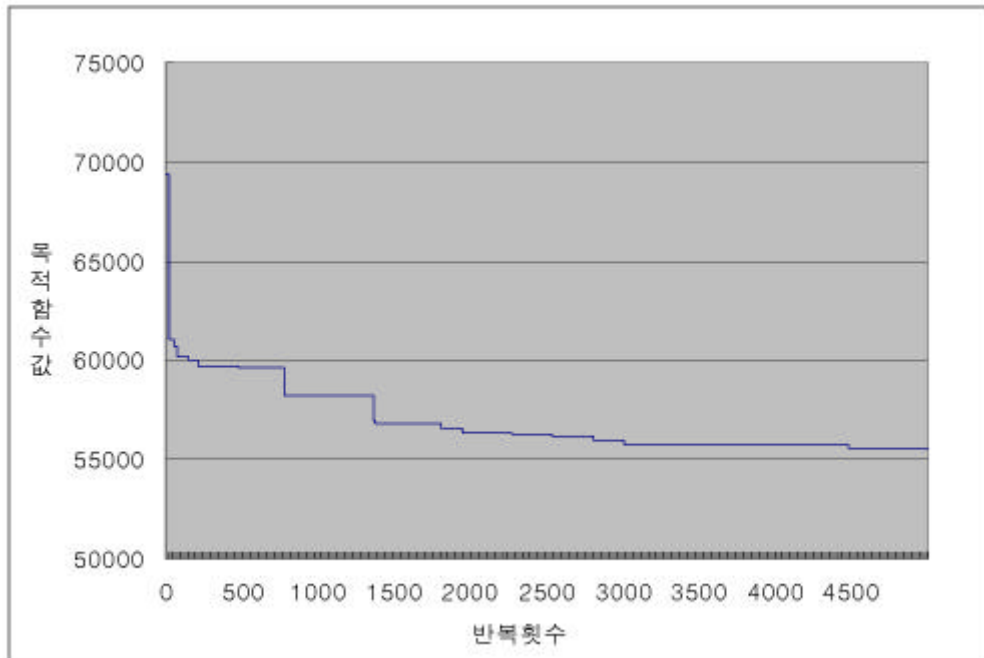


Fig. 4.13  $n=30$ , Type 1 Hybrid GA GA

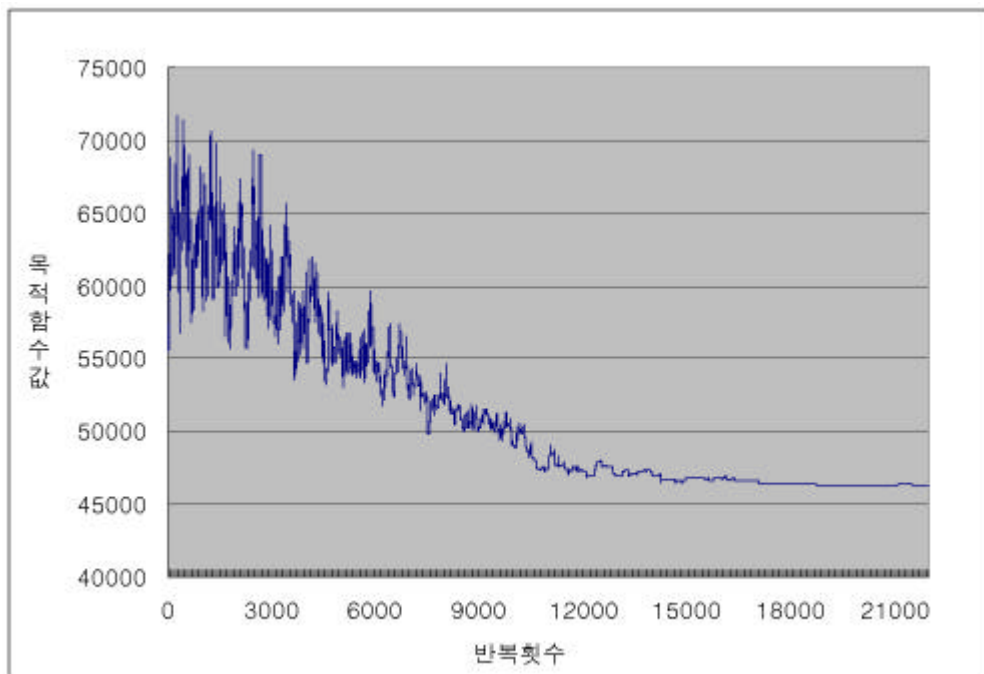


Fig. 4.14  $n=30$ , Type 1 Hybrid GA SA

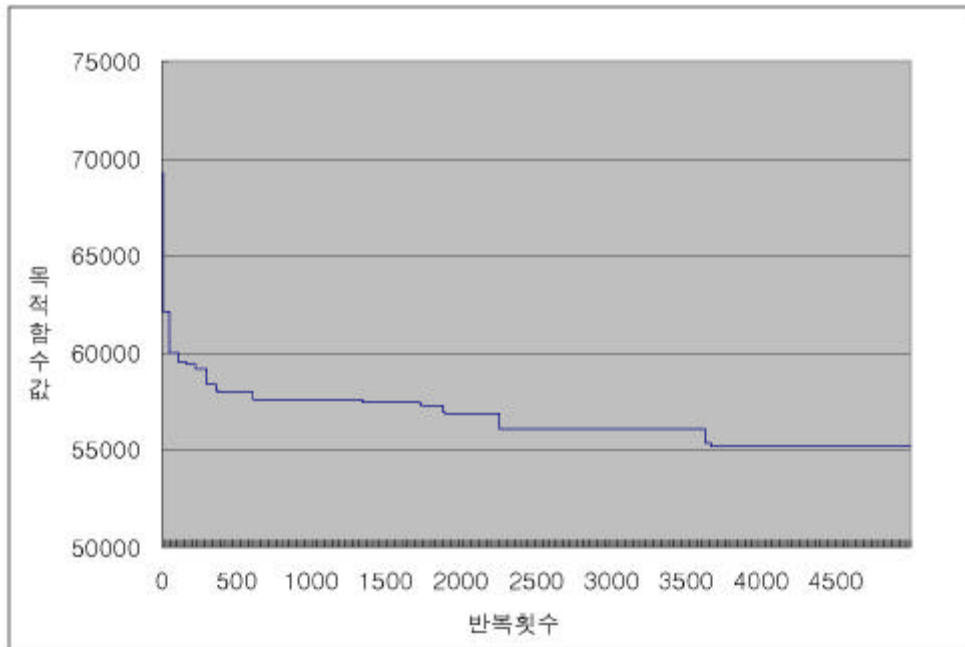


Fig. 4.15  $n=30$ , Type 2 Hybrid GA GA

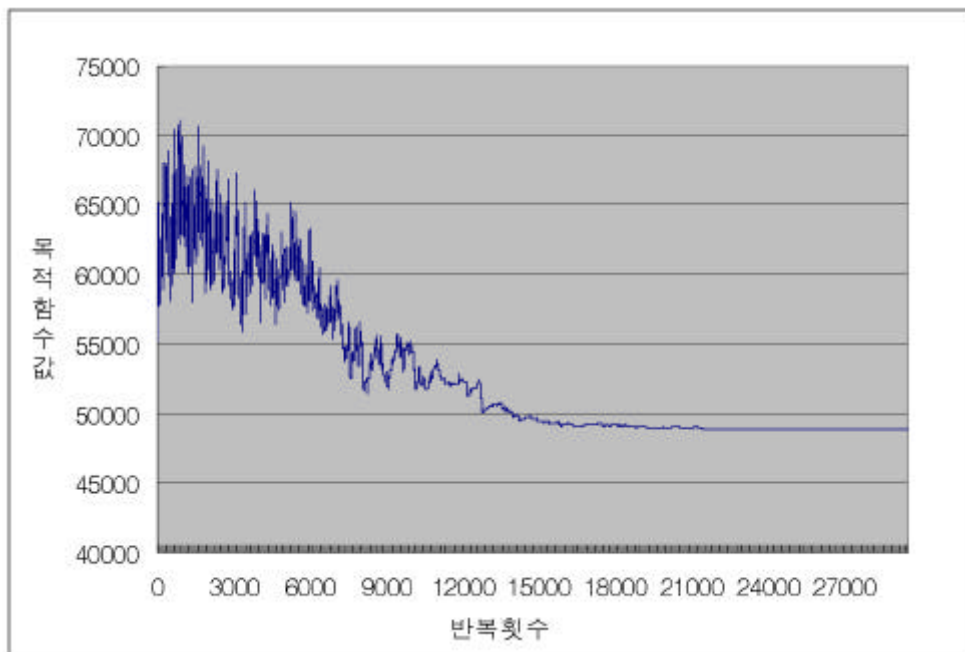


Fig. 4.16  $n=30$ , Type 2 Hybrid GA SA

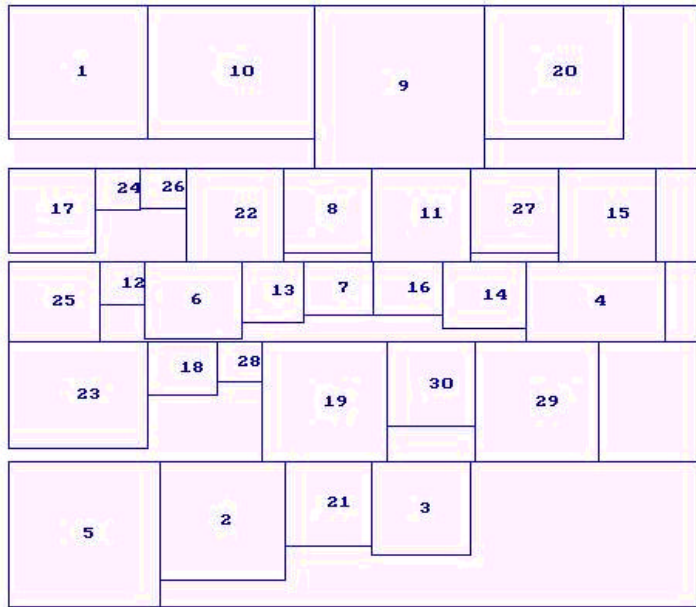


Fig. 4.17          n=30          Type 1

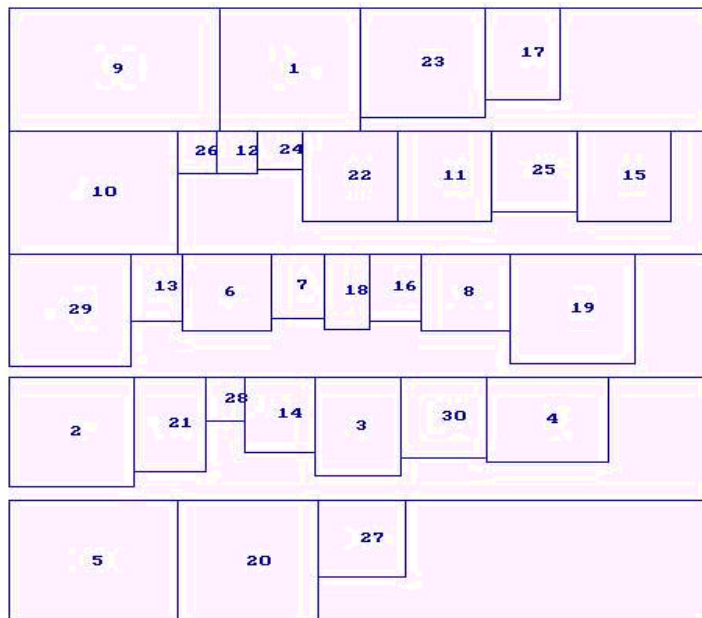


Fig. 4.18          n=30          Type 2

5

가 가  
가  
Hybrid GA

가 GA

가

GA가 Hill-climbing

SA  
가

가

GA SA Hybrid GA

Hybrid GA

, Tam  
and Park GA

17 35%  
2 9%

가 , Woo  
가

20

가

가 .

가

가

.  
, Hybrid GA

(parameter)

가



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1. 가 /

		가 /	가 ,	
1	100	1.0	10.00	10.00
2	80	1.0	8.94	8.94
3	50	1.3	7.07	7.07
4	60	0.8	6.00	10.00
5	120	1.0	10.95	10.95
6	40	1.0	5.71	7.00
7	20	1.4	4.00	5.00
8	40	1.0	6.32	6.32
9	150	1.1	12.24	12.24
10	120	1.5	10.00	12.00
11	50	1.1	7.07	7.07
12	10	1.2	3.16	3.16
13	20	1.5	4.47	4.47
14	30	1.25	5.00	6.00
15	50	1.1	7.07	7.07
16	20	1.5	4.00	5.00
17	40	1.4	6.32	6.32
18	20	1.9	4.00	5.00
19	80	1.0	8.94	8.94
20	100	1.15	10.00	10.00
21	40	1.5	6.32	6.32
22	50	1.1	7.07	7.07
23	80	1.0	8.00	10.00
24	10	1.0	3.16	3.16
25	40	1.1	6.00	6.66
26	10	1.2	3.00	3.33
27	40	1.0	6.32	6.32
28	10	1.3	3.00	3.33
29	80	1.05	8.94	8.94
30	40	1.1	6.32	6.32

2.

(1) 가 5

n	1	2	3	4	5
1	0	5	2	4	1
2	5	0	3	0	2
3	2	3	0	0	0
4	4	0	0	0	5
5	1	2	0	5	0

(2) 가 6

n	1	2	3	4	5	6
1	0	5	2	4	1	0
2	5	0	3	0	2	2
3	2	3	0	0	0	0
4	4	0	0	0	5	2
5	1	2	0	5	0	10
6	0	2	0	2	10	0

(3) 가 7

n	1	2	3	4	5	6	7
1	0	5	2	4	1	0	0
2	5	0	3	0	2	2	2
3	2	3	0	1	0	2	5
4	4	0	1	0	5	2	2
5	1	2	0	5	0	10	0
6	0	2	2	2	10	0	5
7	0	2	5	2	0	5	0

(4) 가 8

n	1	2	3	4	5	6	7	8
1	0	5	2	4	1	0	0	6
2	5	0	3	0	2	2	2	0
3	2	3	0	0	0	0	0	5
4	4	0	0	0	5	2	2	10
5	1	2	0	5	0	10	0	0
6	0	2	0	2	10	0	5	1
7	0	2	0	2	0	5	0	10
8	6	0	5	10	0	1	10	0

(5) 가 9

n	1	2	3	4	5	6	7	8	9
1	0	3	2	0	0	2	10	5	0
2	3	0	4	0	10	4	0	0	2
3	2	4	0	3	4	0	5	5	5
4	0	0	3	0	0	0	0	2	2
5	0	10	4	0	0	5	2	0	0
6	2	4	0	0	5	0	1	2	2
7	10	0	5	0	2	1	0	10	10
8	5	0	5	2	0	2	10	0	1
9	0	2	5	2	0	2	10	1	0

(6) 가 10

n	1	2	3	4	5	6	7	8	9	10
1	0	3	2	0	0	2	10	5	0	5
2	3	0	4	0	10	4	0	0	2	2
3	2	4	0	3	4	0	5	5	5	1
4	0	0	3	0	0	0	0	2	2	0
5	0	10	4	0	0	5	2	0	0	0
6	2	4	0	0	5	0	1	2	2	1
7	10	0	5	0	2	1	0	10	10	5
8	5	0	5	2	0	2	10	0	1	3
9	0	2	5	2	0	2	10	1	0	10
10	5	2	1	0	0	1	5	3	10	0

(7) 가 15

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	10	0	5	1	0	1	2	2	2	2	0	4	0	0
2	10	0	1	3	2	2	2	3	2	0	2	0	10	5	0
3	0	1	0	10	2	0	2	5	4	5	2	2	5	5	5
4	5	3	10	0	1	1	5	0	0	2	1	0	2	5	0
5	1	2	2	1	0	3	5	5	5	1	0	3	0	5	5
6	0	2	0	1	3	0	2	2	1	5	0	0	2	5	10
7	1	2	2	5	5	2	0	6	0	1	5	5	5	1	0
8	2	3	5	0	5	2	6	0	5	2	10	0	5	0	0
9	2	2	4	0	5	1	0	5	0	0	10	5	10	0	2
10	2	0	5	2	1	5	1	2	0	0	0	4	0	0	5
11	2	2	2	1	0	0	5	10	10	0	0	5	0	5	0
12	0	0	2	0	3	0	5	0	5	4	5	0	3	3	0
13	4	10	5	2	0	2	5	5	10	0	0	3	0	10	2
14	0	5	5	5	5	5	1	0	0	0	5	3	10	0	4
15	0	0	5	0	5	0	0	0	2	5	0	0	2	4	0

(8) 가 20

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	5	0	5	2	10	3	1	5	5	5	0	0	5	4	4	0	0	1
2	0	0	3	10	5	1	5	1	2	4	2	5	0	10	10	3	0	5	10	5
3	5	3	0	2	0	5	2	4	4	5	0	0	0	5	1	0	0	5	0	0
4	0	10	2	0	1	0	5	2	1	0	10	2	2	0	2	1	5	2	5	5
5	5	5	0	1	0	5	6	5	2	5	2	0	5	1	1	1	5	2	5	1
6	2	1	5	0	5	0	5	2	1	6	0	0	10	0	2	0	1	0	1	5
7	10	5	2	5	6	5	0	0	0	0	5	10	2	2	5	1	2	1	0	10
8	3	1	4	2	5	2	0	0	1	1	10	10	2	0	10	2	5	2	2	10
9	1	2	4	1	2	1	0	1	0	2	0	3	5	5	0	5	0	0	0	2
10	5	4	5	0	5	6	0	1	2	0	5	5	0	5	1	0	0	5	5	2
11	5	2	0	10	2	0	5	10	0	5	0	5	2	5	1	10	0	2	2	5
12	5	5	0	2	0	0	10	10	3	5	5	0	2	10	5	0	1	1	2	5
13	0	0	0	2	5	10	2	2	5	0	2	2	0	2	2	1	0	0	0	5
14	0	10	5	0	1	0	2	0	5	5	5	10	2	0	5	5	1	5	5	0
15	5	10	1	2	1	2	5	10	0	1	1	5	2	5	0	3	0	5	10	10
16	4	3	0	1	1	0	1	2	5	0	10	0	1	5	3	0	0	0	2	0
17	4	0	0	5	5	1	2	5	0	0	0	1	0	1	0	0	0	5	2	0
18	0	5	5	2	2	0	1	2	0	5	2	1	0	5	5	0	5	0	1	1
19	0	10	0	5	5	1	0	2	0	5	2	2	0	5	10	2	2	1	0	6
20	1	5	0	5	1	5	10	10	2	2	5	5	5	0	10	0	0	1	6	0

(9) 가 30

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	0	3	2	0	0	2	10	5	0	5	2	5	0	0	2	0	5	6	3	0	1	10	0	10	2	1	1	1	0	1
2	3	0	4	0	10	4	0	0	2	2	1	0	5	0	0	0	0	2	0	1	6	1	0	1	2	2	5	1	10	5
3	2	4	0	3	4	0	5	5	5	1	4	1	0	4	0	4	0	6	3	2	5	5	2	1	0	0	3	1	0	2
4	0	0	3	0	0	0	0	2	2	0	6	0	2	5	2	5	1	1	1	1	2	2	4	0	2	0	2	2	5	5
5	0	10	4	0	0	5	2	0	0	0	0	2	0	0	0	0	2	1	0	0	2	0	5	1	0	2	1	0	2	1
6	2	4	0	0	5	0	1	2	2	1	4	10	10	2	5	5	0	5	0	0	0	10	0	0	0	4	0	10	1	1
7	10	0	5	0	2	1	0	10	10	5	10	10	6	0	0	10	2	1	10	1	5	5	2	3	5	0	2	0	1	3
8	5	0	5	2	0	2	10	0	1	3	5	0	0	0	2	4	5	2	10	6	0	5	5	2	5	0	5	5	0	2
9	0	2	5	2	0	2	10	1	0	10	2	1	5	2	0	3	0	2	0	0	4	0	5	2	0	5	2	2	5	2
10	5	2	1	0	0	1	5	3	10	0	5	5	6	0	1	5	5	0	5	2	3	5	0	5	2	10	10	1	5	2
11	2	1	4	6	0	4	10	5	2	5	0	0	0	1	2	1	0	2	0	0	0	6	6	0	4	5	3	2	2	10
12	5	0	1	0	2	10	10	0	1	5	0	0	5	5	2	0	0	0	0	2	0	4	5	10	1	0	0	0	0	1
13	0	5	0	2	0	10	6	0	5	6	0	5	0	2	0	4	2	2	1	0	6	2	1	5	5	0	0	1	5	5
14	0	0	4	5	0	2	0	0	2	0	1	5	2	0	2	1	0	5	3	10	0	0	4	2	0	0	4	2	5	5
15	2	0	0	2	0	5	0	2	0	1	2	2	0	2	0	4	5	1	0	1	0	5	0	2	0	0	5	1	1	0
16	0	0	4	5	0	5	10	4	3	5	1	0	4	1	4	0	0	3	0	2	2	0	2	0	5	0	5	2	5	10
17	5	0	0	1	2	0	2	5	0	5	0	0	2	0	5	0	0	2	2	0	0	0	6	5	3	5	0	0	5	1
18	6	2	6	1	1	5	1	2	2	0	2	0	2	5	1	3	2	0	5	1	2	10	10	4	0	0	5	0	0	0
19	3	0	3	1	0	0	10	10	0	5	0	0	1	3	0	0	2	5	0	0	5	5	1	0	5	2	1	2	10	10
20	0	1	2	1	0	0	1	6	0	2	0	2	0	10	1	2	0	1	0	0	5	2	1	3	1	5	6	5	5	3
21	1	6	5	2	2	0	5	0	4	3	0	0	6	0	0	2	0	2	5	5	0	4	0	1	0	0	0	5	0	0
22	10	1	5	2	0	10	5	5	0	5	6	4	2	0	5	0	0	10	5	2	4	0	5	0	4	4	5	0	2	5
23	0	0	2	4	5	0	2	5	5	0	6	5	1	4	0	2	6	10	1	1	0	5	0	0	4	4	1	0	2	2
24	10	1	1	0	1	0	3	2	2	5	0	10	5	2	2	0	5	4	0	3	1	0	0	0	5	5	0	1	0	0
25	2	2	0	2	0	0	5	5	0	2	4	1	5	0	0	5	3	0	5	1	0	4	4	5	0	1	0	10	1	0
26	1	2	0	0	2	4	0	0	5	10	5	0	0	0	0	0	5	0	2	5	0	4	4	5	1	0	0	0	0	0
27	1	5	3	2	1	0	2	5	2	10	3	0	0	4	5	5	0	5	1	6	0	5	1	0	0	0	0	0	0	10
28	1	1	1	2	0	10	0	5	2	1	2	0	1	2	1	2	0	0	2	5	5	0	0	1	10	0	0	0	2	2
29	0	10	0	5	2	1	1	0	5	5	2	0	5	5	1	5	5	0	10	5	0	2	2	0	1	0	0	2	0	2
30	1	5	2	5	1	1	3	2	2	2	10	1	5	5	0	10	1	0	10	3	0	5	2	0	0	0	10	2	2	0

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