

工學碩士 學位論文

Hybrid GA

A Study on Optimal Facility Layout of Block Facility
using Hybrid GA

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2001年 2月

韓國海洋大學校 大學院

物流 工學科

李 容 煜

1.	1
1.1	1
1.2	2
1.3	4
2.	5
2.1	5
2.2	7
3.	11
3.1	11
3.2	13
3.3	(Genetic Algorithm)	15
3.3.1	17
3.3.2	가	18
3.3.3	18
3.3.4	23
3.4	(Simulated Annealing)	24
3.3.1	(T)	26
3.3.2	(Perturbation)	26
3.3.3	27
3.3.4	27

3.5	Hybrid GA	28
3.5.1	(GA)	28
3.5.2	(SA)	29
4.	가	31
4.1	Parameter	31
4.1.1	GA Parameter	31
4.1.2	SA Parameter	32
4.2		33
5.		44
		46
		49

List of Tables

Table 1.1	3
Table 4.1	Parameter	32
Table 4.2	5 10 , Hybrid GA	33
Table 4.3	15, 20, 30 , Hybrid GA	34

List of Figures

Fig. 2.1	6
Fig. 2.2	2가	10
Fig. 3.1 Hybrid GA	14
Fig. 3.2	16
Fig. 3.3	17
Fig. 3.4 (PMX)	19
Fig. 3.5 (OX)	20
Fig. 3.6 (CX)	20
Fig. 3.7 Inversion	21
Fig. 3.8 Insertion	22
Fig. 3.9 Swap	22
Fig. 3.10	25
Fig. 3.11 Hybrid GA	Flowchart	30
Fig. 4.1 가 n=15 , Type 1 Hybrid GA GA	35
Fig. 4.2 가 n=15 , Type 1 Hybrid GA SA	35
Fig. 4.3 가 n=15 , Type 2 Hybrid GA GA	36
Fig. 4.4 가 n=15 , Type 2 Hybrid GA SA	36
Fig. 4.5 n=15 Type 1	37
Fig. 4.6 n=15 Type 2	37

Fig. 4.7	가 n=20	, Type 1	Hybrid GA	GA	38
.....					
Fig. 4.8	가 n=20	, Type 1	Hybrid GA	SA	38
.....					
Fig. 4.9	가 n=20	, Type 2	Hybrid GA	GA	39
.....					
Fig. 4.10	가 n=20	, Type 2	Hybrid GA	SA	39
.....					
Fig. 4.11	n=20	Type 1			40
Fig. 4.12	n=20	Type 2			40
Fig. 4.13	가 n=30	, Type 1	Hybrid GA	GA	41
.....					
Fig. 4.14	가 n=30	, Type 1	Hybrid GA	SA	41
.....					
Fig. 4.15	가 n=30	, Type 2	Hybrid GA	GA	42
.....					
Fig. 4.16	가 n=30	, Type 2	Hybrid GA	SA	42
.....					
Fig. 4.17	n=30	Type 1			43
Fig. 4.18	n=30	Type 2			43

A Study on Optimal Facility Layout of Block Facility using Hybrid GA

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Abstract

Facility layout is the early stage of system design that requires a mid-term or long-term plan. Since improper facility layout might incur substantial logistics cost including material handling and re-installment costs, due consideration must be given to decisions on facility layout.

Facility layout is concerned with how to arrange equipment necessary for production in a given space. Its objective is to minimize the sum of all the products of each equipment's amount of flow multiplied by distance. Facility layout also is related to the issue of NP-complete, i.e., calculated amounts exponentially increase with the increase of the number of equipment.

This study discusses Hybrid GA developed, as an algorithm for facility layout, to solve the above-mentioned problems. The Algorithm, which is designed to efficiently place equipment, automatically produces a horizontal passageway by the block, if a designer provides the width and length of the space to be handled. In

addition, this study demonstrates the validity of the Algorithm by comparing with existing algorithms that have been developed.

We present a hybrid GA approach to the facility layout problem that improves on existing work in terms of solution quality and method.

Experimental results show that the proposed algorithm is able to produce better solution quality and more practical layouts than the ones obtained by applying existing algorithms.

1

1.1

, 가 가

· ,

,

·

1955 , GNP 8% 가

20 50%가

·

10 30%

, 15 가 3

가 ,

,

·

가

가 가 가 가

Hybrid GA(Genetic Algorithm)

1.2

가 Muther (System Layout Planning)
가

가 (Quadratic Assignment Problem : QAP)

QAP

20 Job shop 608 가

가 가

(construction algorithm)

가

(improvement algorithm),

(hybrid algorithm), (graph theoretic algorithm) [29].

	ALDEP	Seehof and Evans 1967	
	CORELAP	Lee and Moore 1967	
	MAT	Edward et al. 1970	
	FATE	Block 1978	MAT FATE
	FLAT	Heragu and Kusiak 1986	3
	CRAFT	Armour and Buffar 1963	
	FLAT	Khalil 1973	
	MULTIPLE	Bozer et al. 1994	Spacefilling Curve
	FLAC	Scriabin and Vergin 1995	, FLAT
	BLOCKPLAN	Donaghey and Pire	2-opt

Table 1.1

Tate Smith가 1995 aspect ratio
[28].
1995 Suresh et al 가
[22].
QAP Crossover .
1996 GA X GA
CRAFT [12], 1997
Bay [2].
1997
[1].

1.3

1 ,
2 .
3 .
4 가 .
, 5 .

2.1

(facilities layout)

, ,
가 [13].

(facilities layout problem)

가 .

Fig. 2.1

가 . (facilities location)

(facilities design)

,
가
(facilities

system design)

(handling system design),

(layout design)

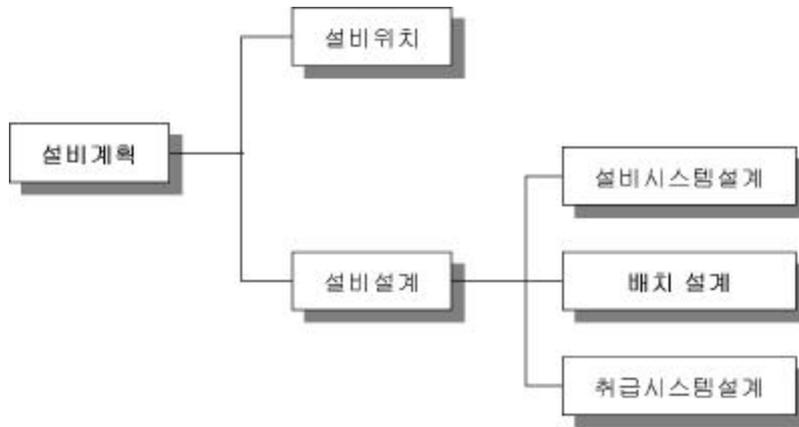


Fig. 2.1 (Tomkins et al. 1996)

가
 가
 가
 가 NP-complete
 가
 가
 가
 (2.1)

$$\text{Min } F(x) = \sum_{i=1}^n \sum_{j=1}^n C_i \cdot a_{ij} \cdot d_{ij}, \quad i, j = 1, 2, 3, \dots, n \quad (2.1)$$

$$\begin{aligned}
 a_{ij} &= \text{ } i \text{ } j \\
 C_i &= \text{ } \\
 d_{ij} &= \text{ } i \text{ } j
 \end{aligned}$$

$$C_t = 1, \quad a_{ij} = d_{ij}$$

()

가

2.2

2.2.1

가 가

가

가

가

가

가

가

. Fig. 2.2

가

2.2.2

가 가

가 /

가

가

Hybrid GA

() (Type 1)

: 가 가

가 가

()

가 가

가

가 가

가

가

() (Type 2)

: 가 /

가 / 가 가

가

가 가 /

가 /

가 가

가 /

가

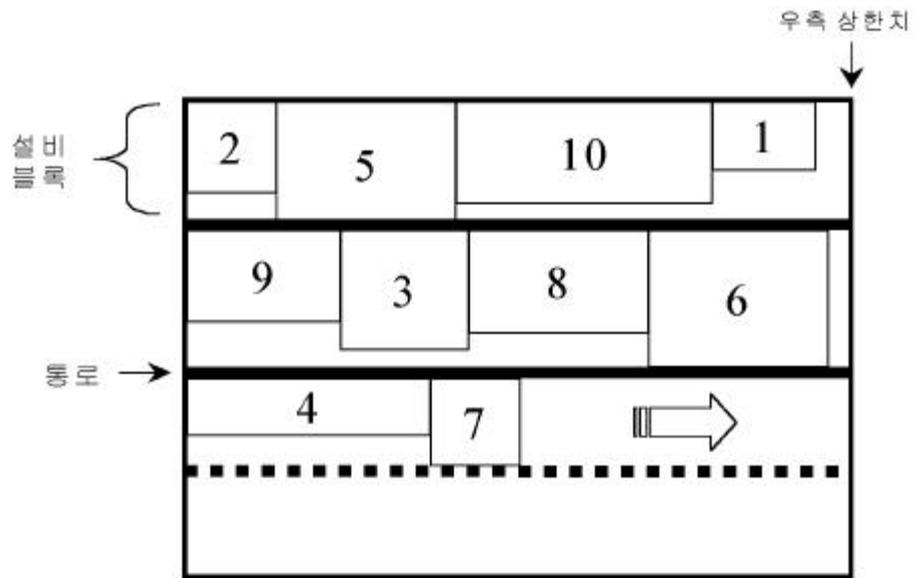
가 /

가 / 가
가
가 /

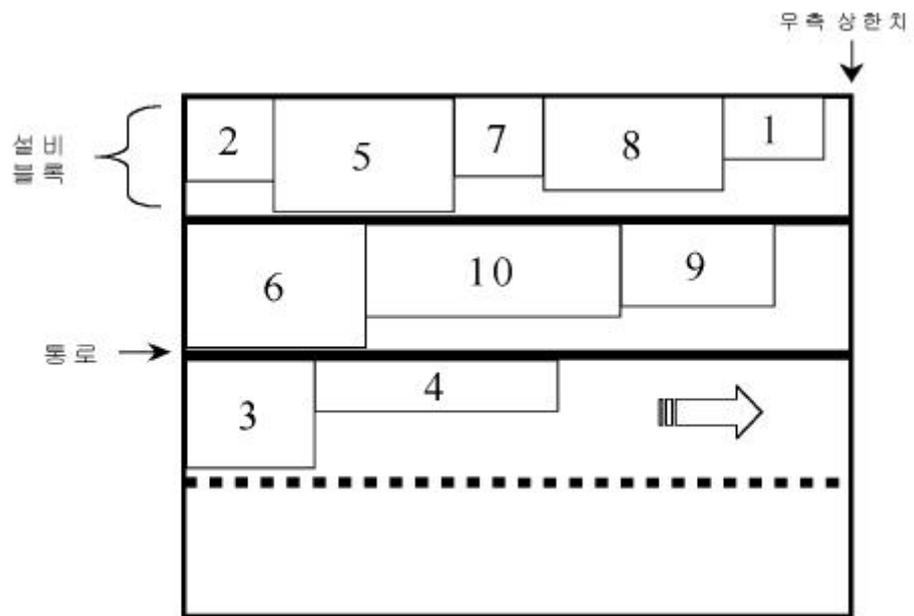
가 가
가
가 ,

가

[1].



: Type 1 ()



: Type 2 ()

Fig. 2.2 2가

3

3.1

가 .
1950 1960
1947 Dantzig
1970 , 가
TSP(Traveling Salesman Problem) 1970
가 .
NP-complete
NP-complete
가 ,
가
Genetic Algorithm, Simulated Annealing,
가 15
가 NP-Complete
가 가 가

가 , 가 가

가

가

가

(Neural network)
(neuron)

가 ,

가 가

(Genetic algorithm) 가

가 ,

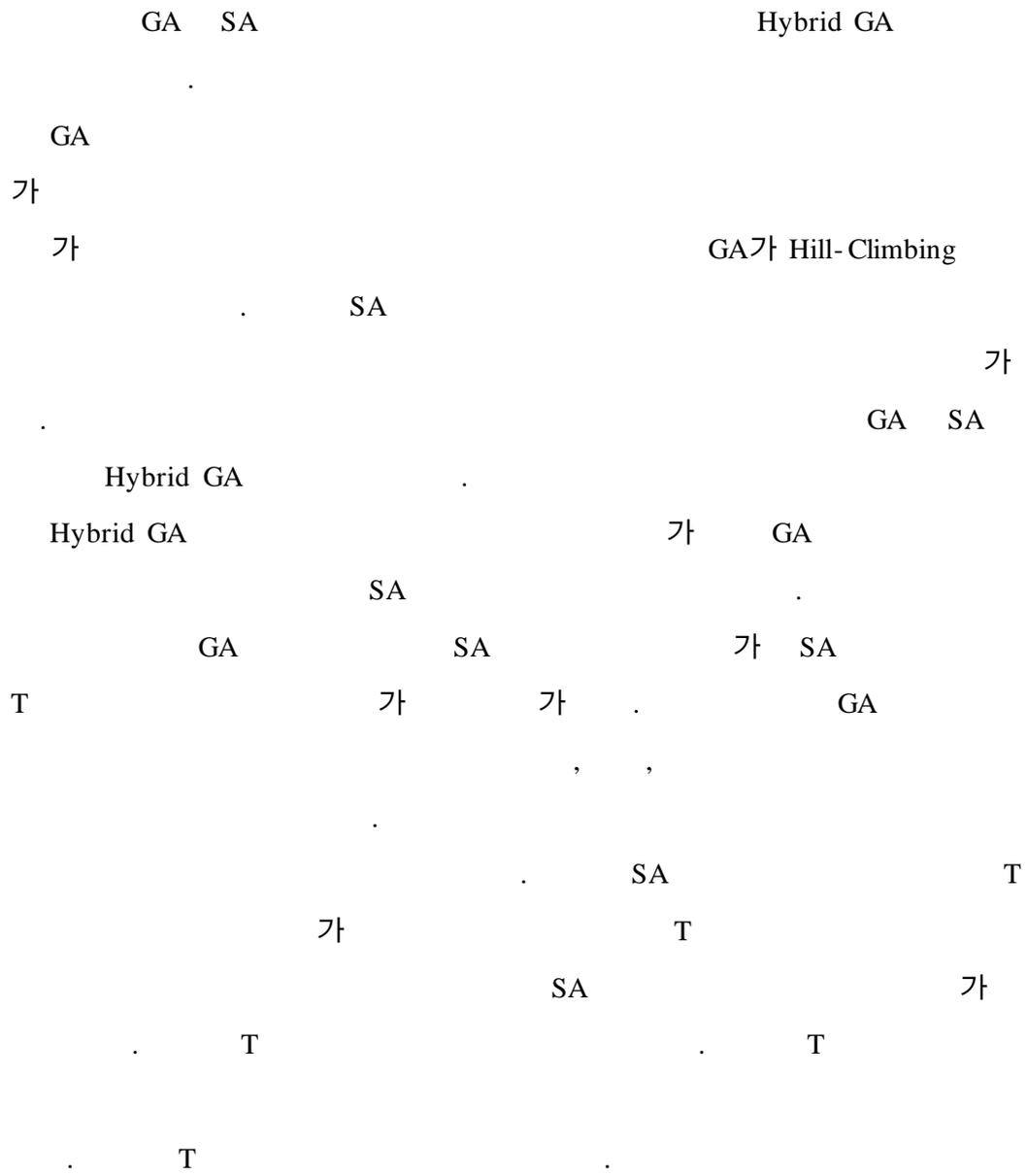
가

가

(Simulated annealing) (Iterative improve-
ment)

가

가



3.2

- (3.2)
- 1 : , , , 가
 - 2 :
 - 3 : 2 Genetic Algorithm
 - 4 : 5 가 , 3 가
Genetic Algorithm
 - 5 : Simulated Annealing
 - 6 : 5 가
Simulated Annealing

Fig. 3.1

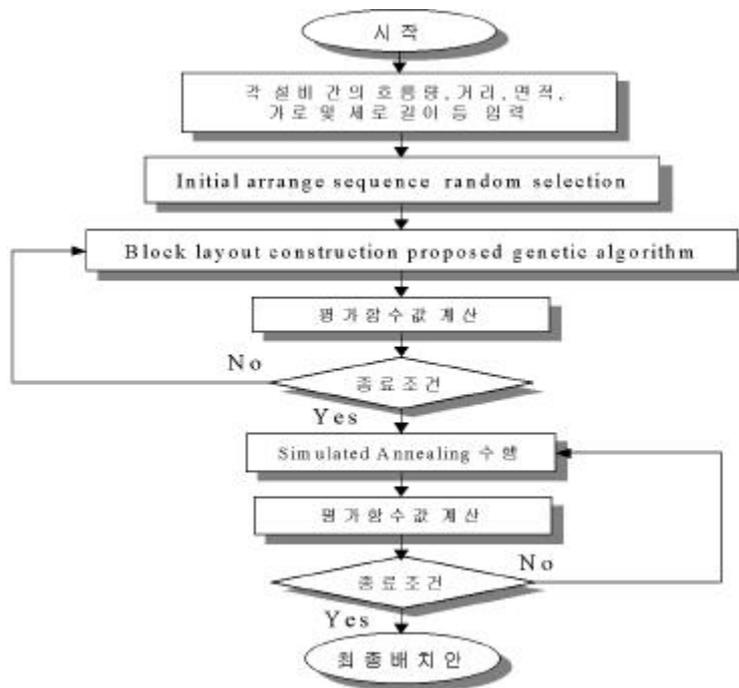


Fig. 3.1 Hybrid GA

3.3

(Genetic Algorithm)

(Genetic Algorithm, GA)

. GA

(natural selection)

(survival of the fittest)

GA

가

. GA

(chromosome)

가

(reproduction),

(crossover),

(mutation)

가

가

GA가

, GA

(coding)

(point)

가

(fitness function)

. , GA

. , GA 가

(multimodal)

가

GA

GA

Fig. 3.2

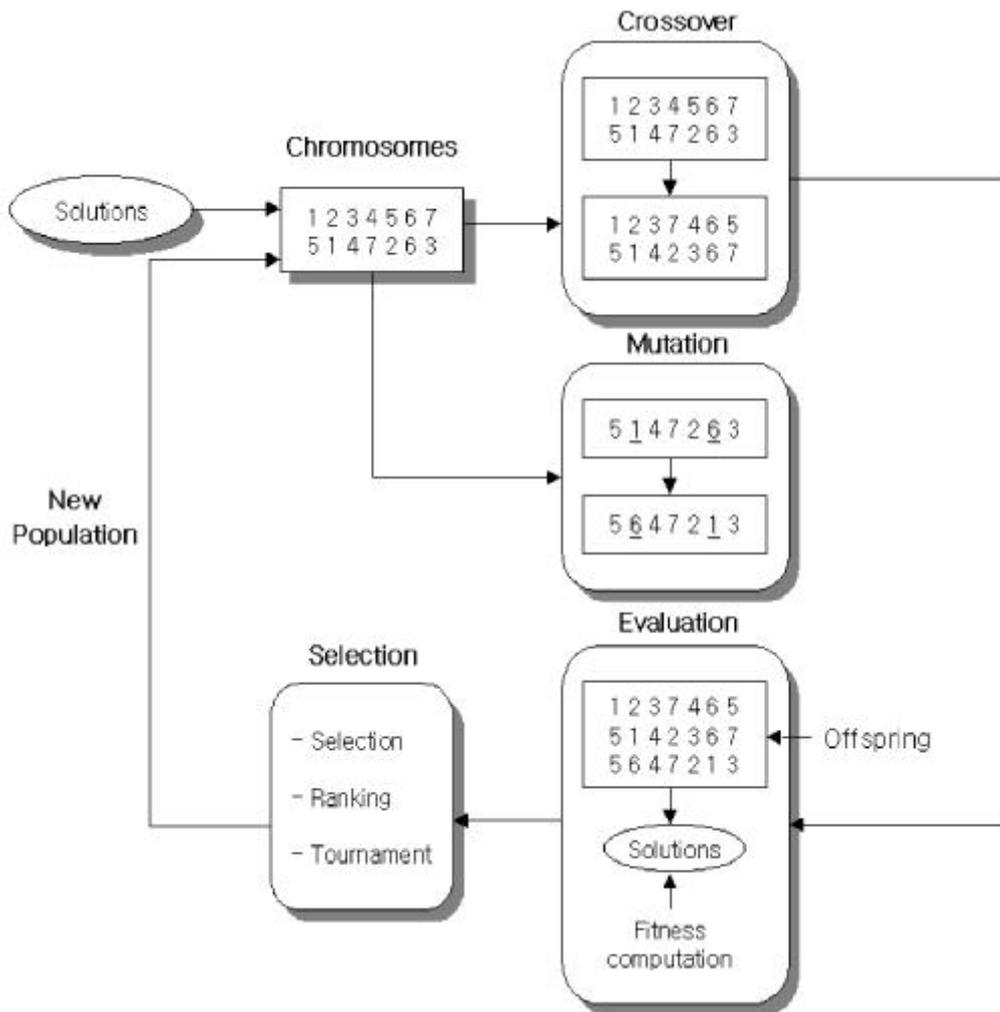


Fig. 3.2

3.3.2 가

가

$$\text{Min } F(x) = \sum_{i=1}^n \sum_{j=1}^n C_i \cdot a_{ij} \cdot d_{ij}, \quad i, j = 1, 2, 3, \dots, n$$

$$a_{ij} = \quad i \quad j$$

$$C_i =$$

$$d_{ij} = \quad i \quad j$$

$$C_i = 1, \quad a_{ij} \quad d_{ij}$$

()

가

3.3.3

(crossover),

(mutation)가

(selection),

(1) (crossover)

(crossover)

가

가

2

(character preservingness)

(crossover neighborhood)

(PMX : Partially matched crossover)

TSP

A B

2

(cut - point)

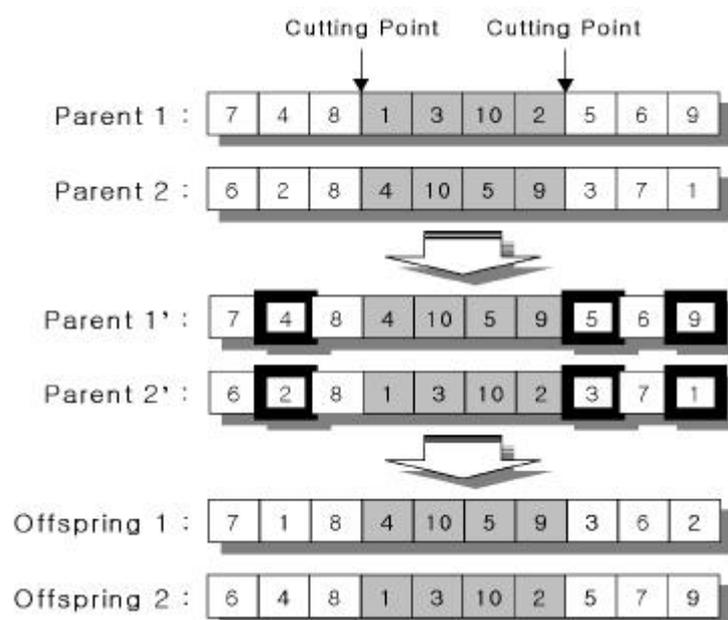


Fig. 34

(PMX)

(OX : ordered crossover)

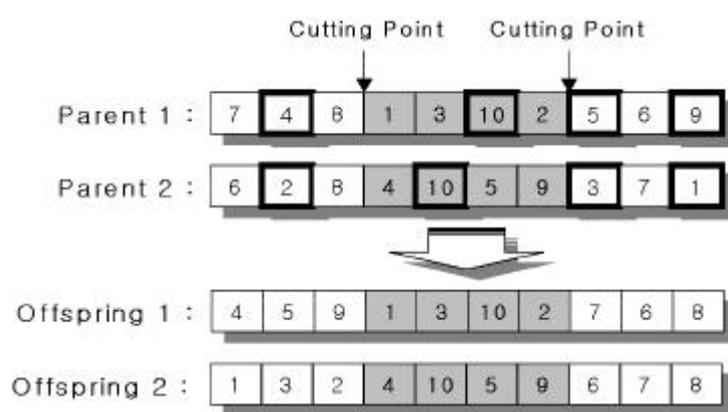


Fig. 3.5 (OX)

(CX : cycle crossover)

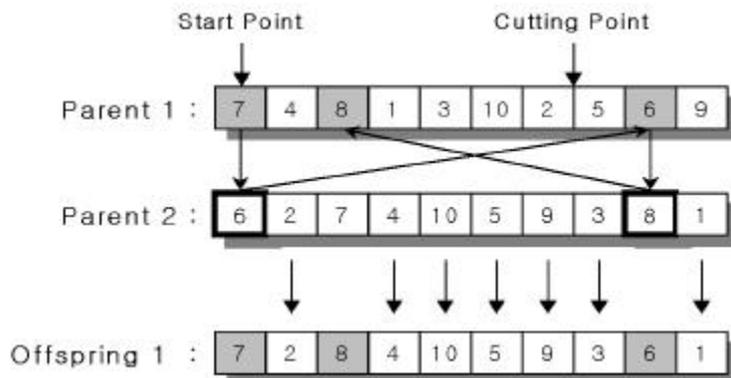


Fig. 3.6 (CX)

(2) (mutation)

가

가

가

가

Inversion, Insertion,

Swap

Inversion

(cutting point)

가

가

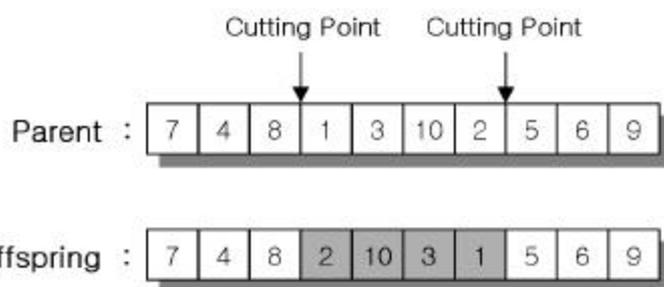


Fig. 3.7 Inversion

Insertion

string

가

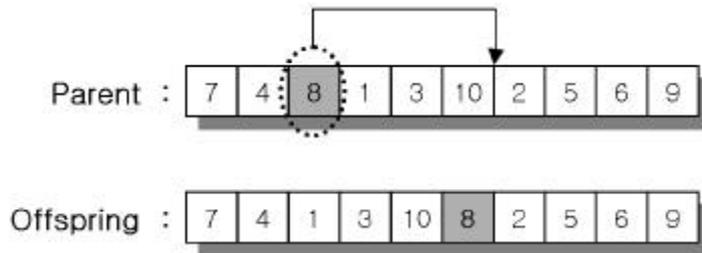


Fig. 3.8 Insertion

Swap

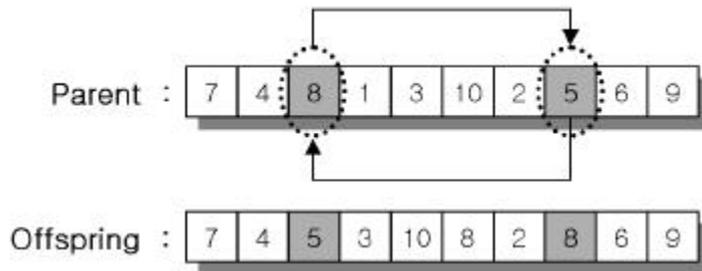


Fig. 3.9 Swap

(3) (selection)

(natural selection)

(ranking selection)

(tournament)

3.4

(Simulated Annealing)

(Simulated Annealing, SA)

가 .
가 .
가 .
가 .
T 가 .
가 .
가 .
가 .
SA 가 .
(Annealing Schedule)
SA Hill-Climbing Steepest Descent . ,
SA .
Metropolis
Metropolis

Hill Climbing Steepest Descent

가

Metropolis

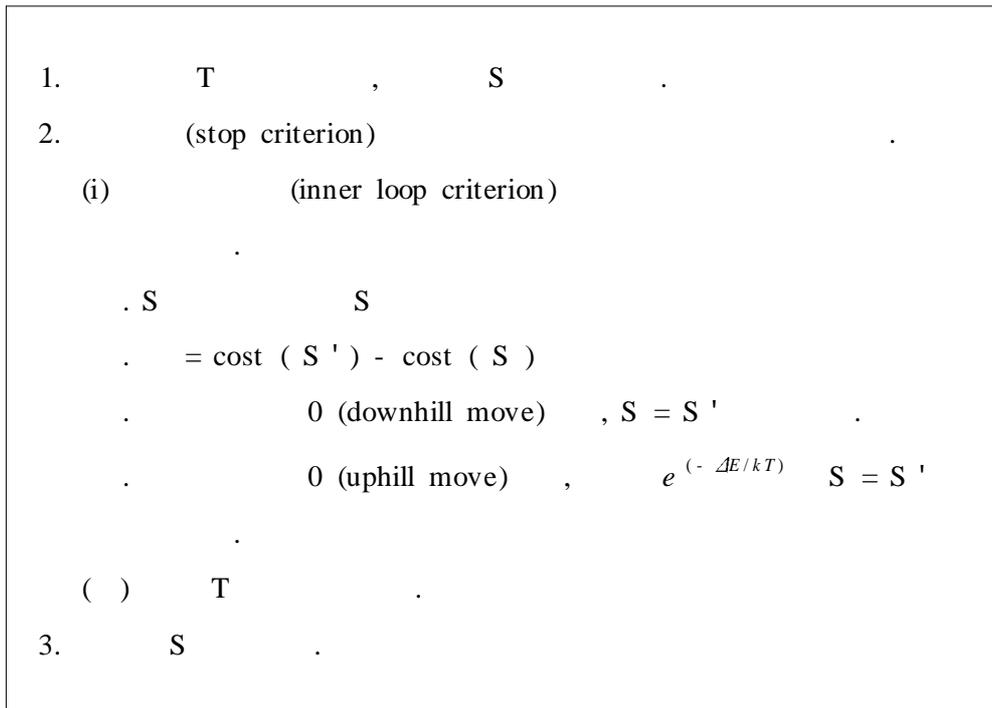


Fig. 3.10

SA

가

가

Metropolis

Metropolis

SA

가 ,

,
,

가

Hybrid GA

GA

가

string

가 SA

3.4.1 (T)

Hybrid GA

T

$$\exp(-\Delta E/kT) \cong 1$$

가

가

(T) 5

3.4.2 (Perturbation)

Fig. 3.10

2()

S'

가

가
(mutation)
string

3.4.3

L
L , 가 L
가

Fig. 3.10 2 () (3.1)

$$T' = r \cdot T \quad (3.1)$$

T' , r 0.95 0.99

3.4.4

Fig. 3.10 2

, 가
() ,

가

T

가

3.5 Hybrid GA

3.5.1 (Genetic Algorithm)

- [0] . , , 가 / , 가 , population , 가 .
- [1] Population string .
- [2] string (crossover) .
- [3] (mutation) .
- [4] 가 .
- [5] Reproduction Rule .
- [6] [7] , , [3] 가 .
- [7]

3.5.2

(Simulated Annealing)

[1] GA 가 , T
L .

[2] .

[3] (inner loop criterion)

[3-1] GA 가 .

[3-2] GA 가 .

[3-3] 가 0 ,
 , $e^{(-\Delta/kT)}$.

[4] T .

[5] [6] ,

, [3] 가 .

[6]

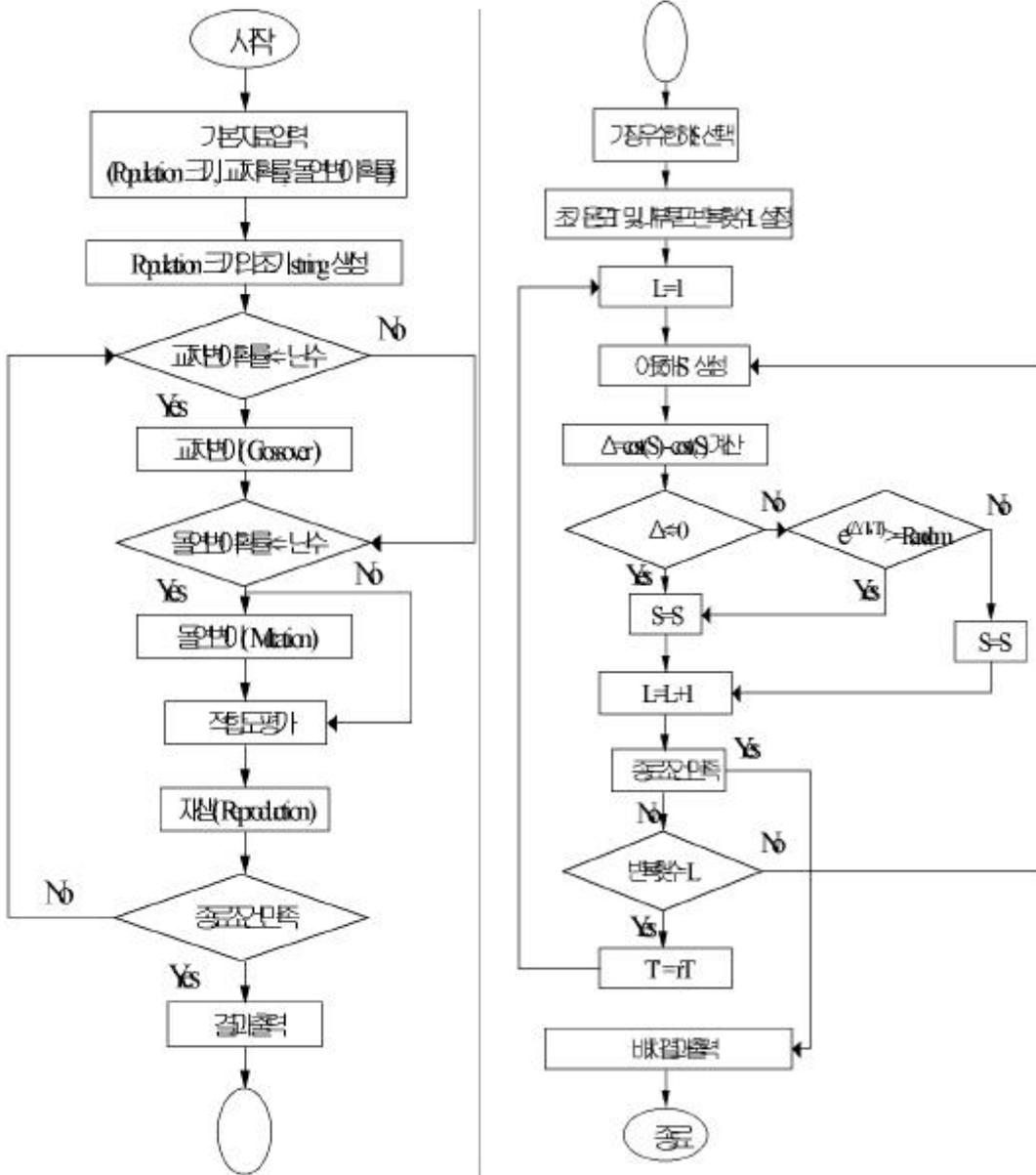


Fig. 3.11 Hybrid GA

Flowchart

4 가

Hybrid GA

가 5, 6, 7, 8, 9, 10, 15, 20, 30

Tam GA

(

[1][15][21]).

Hybrid GA Visual C++ , 450Mhz, RAM
128 Mbyte, Windows 98 가 PC .

4.1 Parameter

4.1.1 Genetic Algorithm Parameter

parameter .

Population size 10 100 10 가 50
50 50
50 Population size 50

. Crossover rate Mutation rate 0.0 1.0 0.05 가

Crossover rate 0.2 0.3 , Mutation rate 0.1

. 가 parameter

Population size 50 가 $n = 20$ Crossover

Rate = 0.1 0.3, Mutation Rate = 0.1 Crossover Rate 0.01

, Population size = 50, Crossover Rate = 0.25, Mutation Rate = 0.1

가 generation 5000

, 가 2000 가 .

4.1.2 Simulated Annealing Parameter

가 .

20 (T) 5

500 . (r) 0.9, 0.5

.

,

, 가

가 . ,

가 , (Boltzmann factor)

$$\exp\left(-\frac{\Delta E}{K_B \cdot T}\right)$$

. (ΔE)가 ,

K_B 가 .

(n)			(K_B)	
15	5	0.9	80 90	500
20	5	0.9	150 160	500
30	5	0.9	300	500

Table 4.1 Parameters

4.2

가 가 5 10

Table 4.2

가 5 10

가

가

(n)			GA (Woo and Park)	Hybrid GA	String	
5	Type 1	512	512	512	1-2-3-4-5	100%
	Type 2	533	533	533	2-1-3-5-4	100%
6	Type 1	896	896	896	1-2-3-4-5-6	100%
	Type 2	884	884	884	3-2-5-1-4-6	100%
7	Type 1	1319	1319	1319	1-4-5-2-3-7-6	100%
	Type 2	1230	1230	1230	1-4-5-2-3-7-6	100%
8	Type 1	1940	1965	1940	1-2-5-8-7-4-6-3	100%
	Type 2	1806	1806	1806	5-1-2-6-4-8-7-3	100%
9	Type 1	2673	2673	2673	1-2-5-8-7-3-6-4-9	100%
	Type 2	2664	2664	2664	1-2-5-8-7-3-6-9-4	100%
10	Type 1	3709	3709	3709	1-2-5-8-7-3-6-10-9-4	100%
	Type 2	3519	3591	3519	10-9-1-8-7-3-4-6-2-5	100%

Table 4.2 가 5 10 , Hybrid GA

PC 15 30

Table 4.3

, Tam 17 35% 가 , Woo and
 Park GA 2 9% 가 .
 , 20
 , 가
 가 Hill-Climbing

SA GA

가

(n)		Tam	Woo & Park GA	Hybird GA		Efficiency of Hybrid GA	
						Tam	Woo & Park
15	Type 1	13762	9120	8847	8911	35.71%	2.99%
	Type 2	12240	9855	9521	9582	22.21%	3.39%
20	Type 1	26921	21885	20029	20696	25.60%	8.48%
	Type 2	28646	22656	21698	21926	24.25%	4.23%
30	Type 1	55668	50492	46286	46545	18.06%	9.66%
	Type 2	58824	52884	48814	49324	17.02%	7.70%

Table 4.3 가 15, 20, 30 , Hybrid GA

Hybrid GA

Type 1, Type 2

$n = 15, 20, 30$, GA

SA

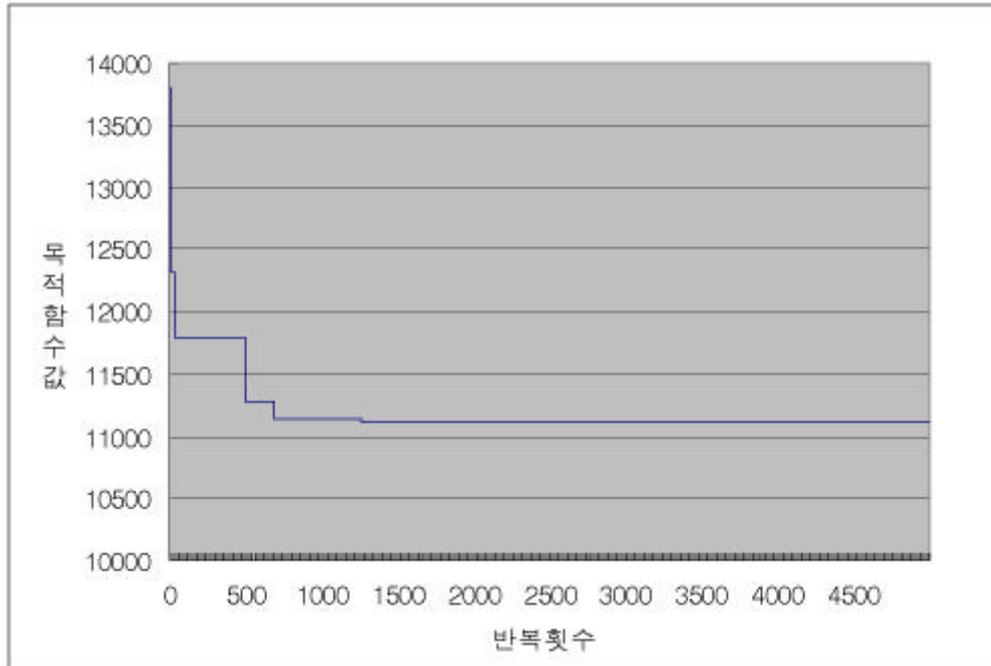


Fig. 4.1 n=15, Type 1 Hybrid GA GA

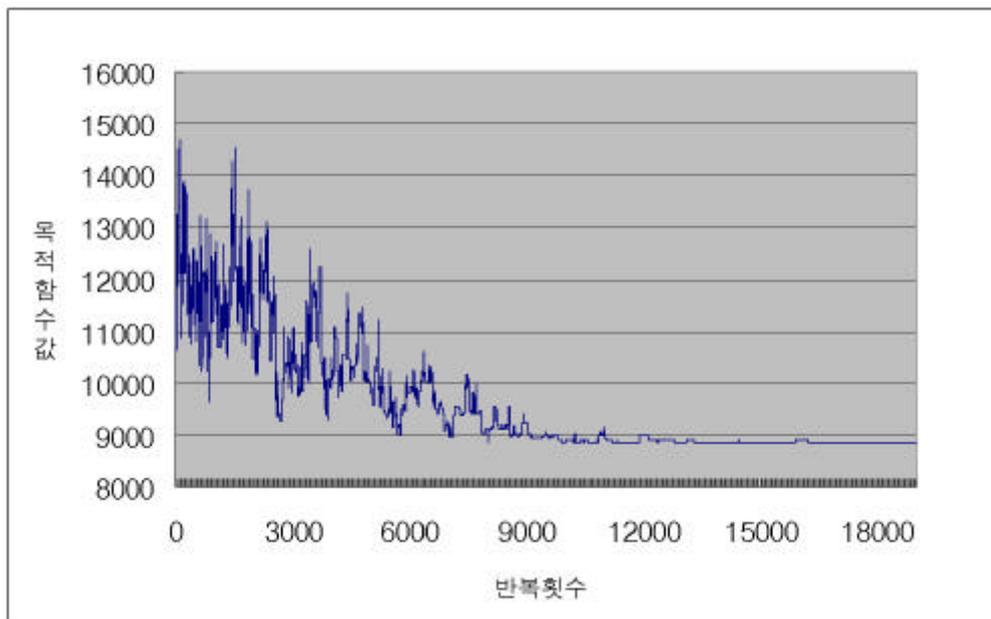


Fig. 4.2 n=15, Type 1 Hybrid GA SA

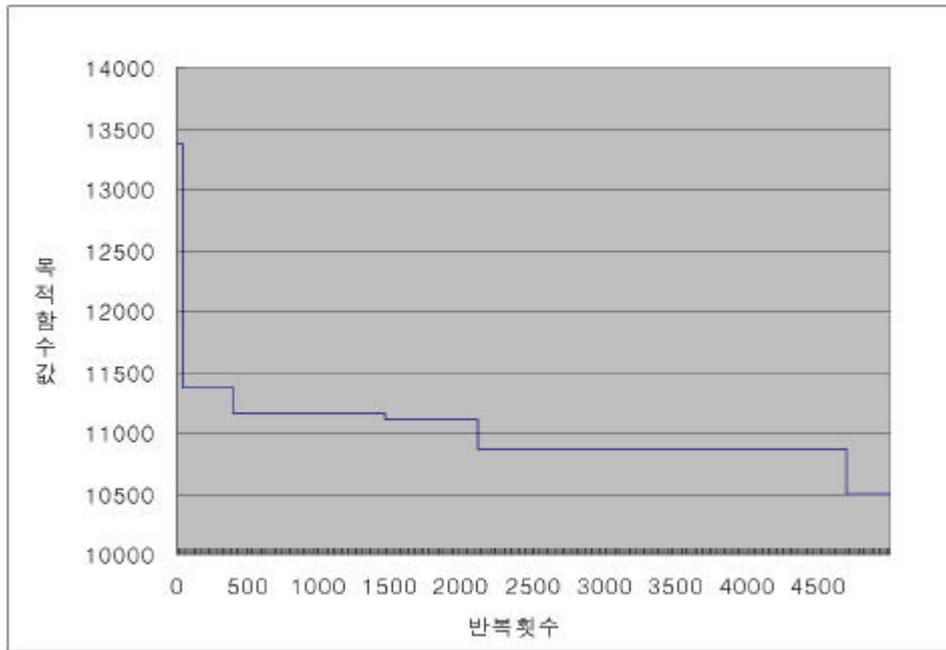


Fig. 4.3 n=15 , Type 2 Hybrid GA GA

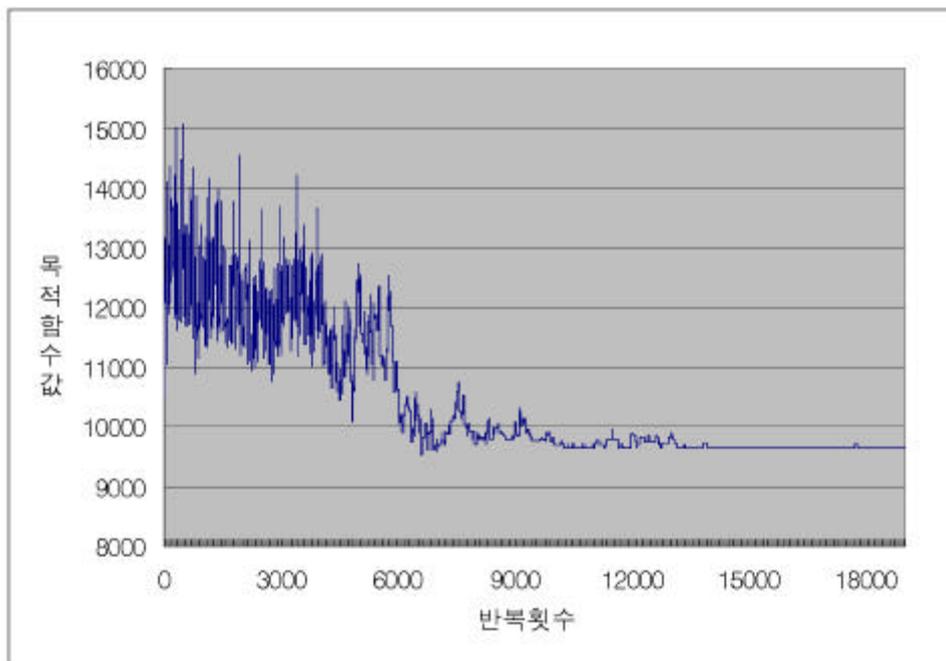


Fig. 4.4 n=15 , Type 2 Hybrid GA GA

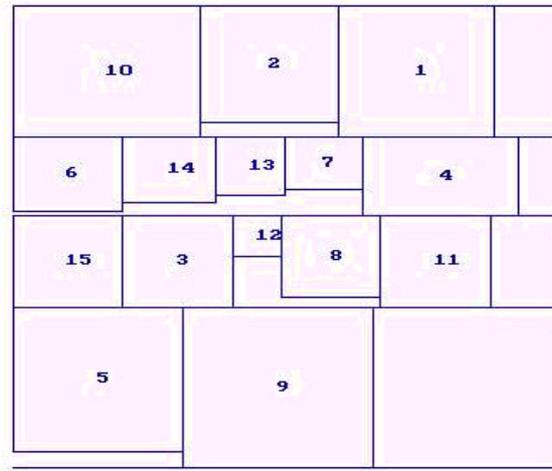


Fig. 4.5 n=15 Type 1

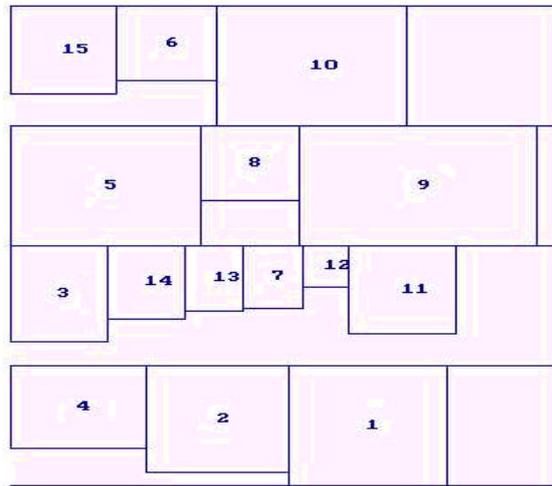


Fig. 4.6 n=15 Type 2

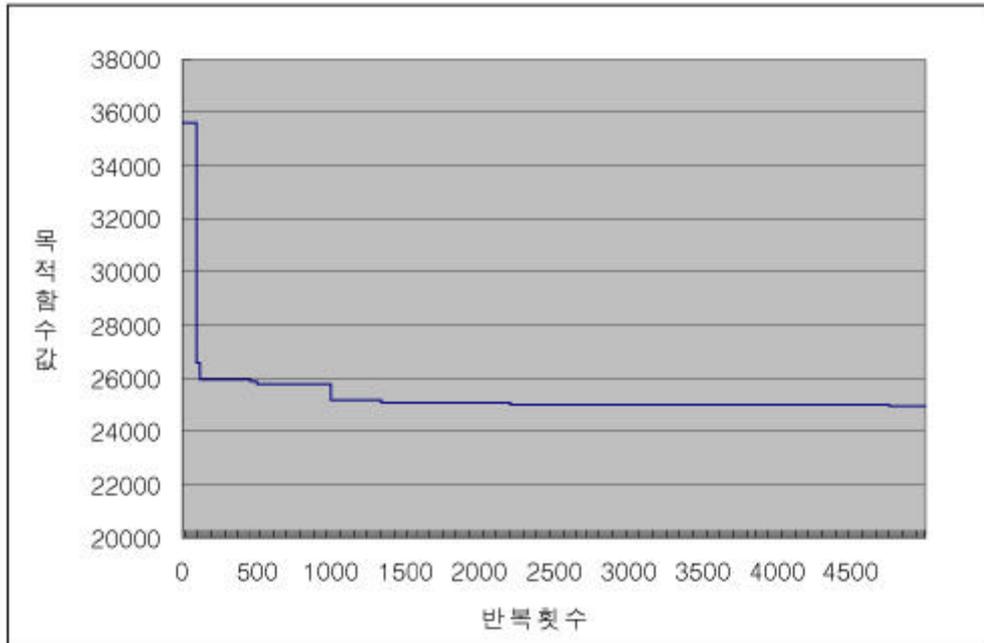


Fig. 4.7 n=20 , Type 1 Hybrid GA GA

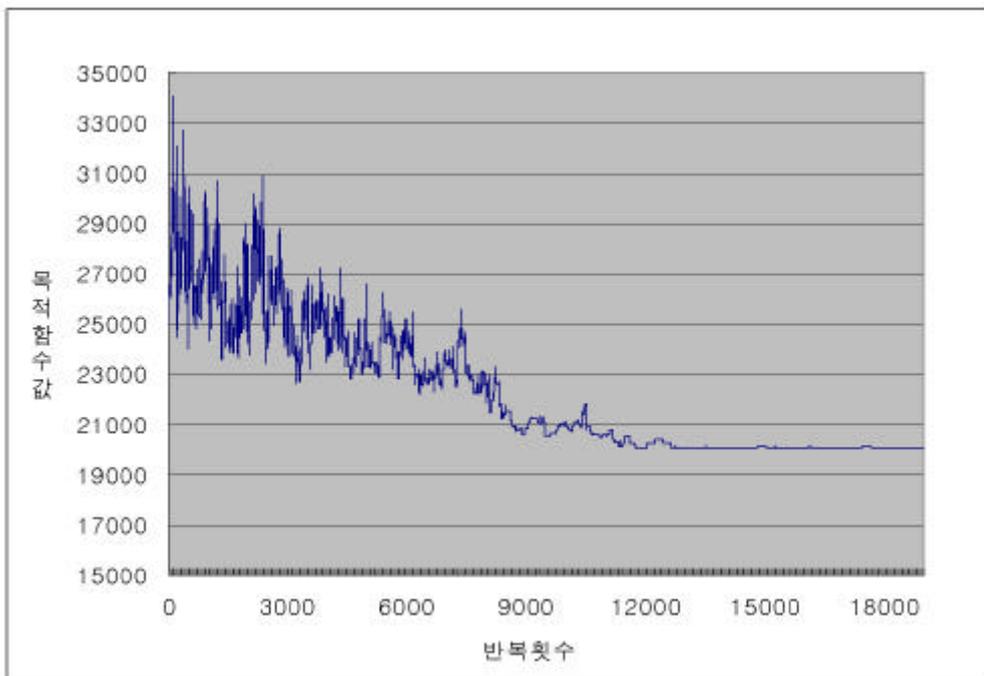


Fig. 4.8 n=20 , Type 1 Hybrid GA SA

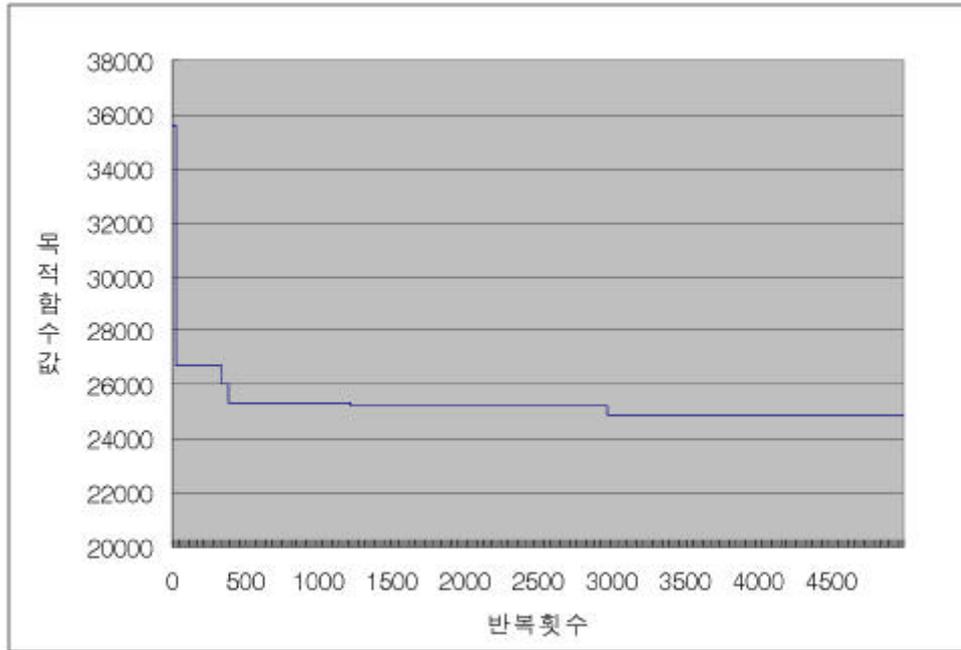


Fig. 4.9 $n=20$, Type 2 Hybrid GA GA

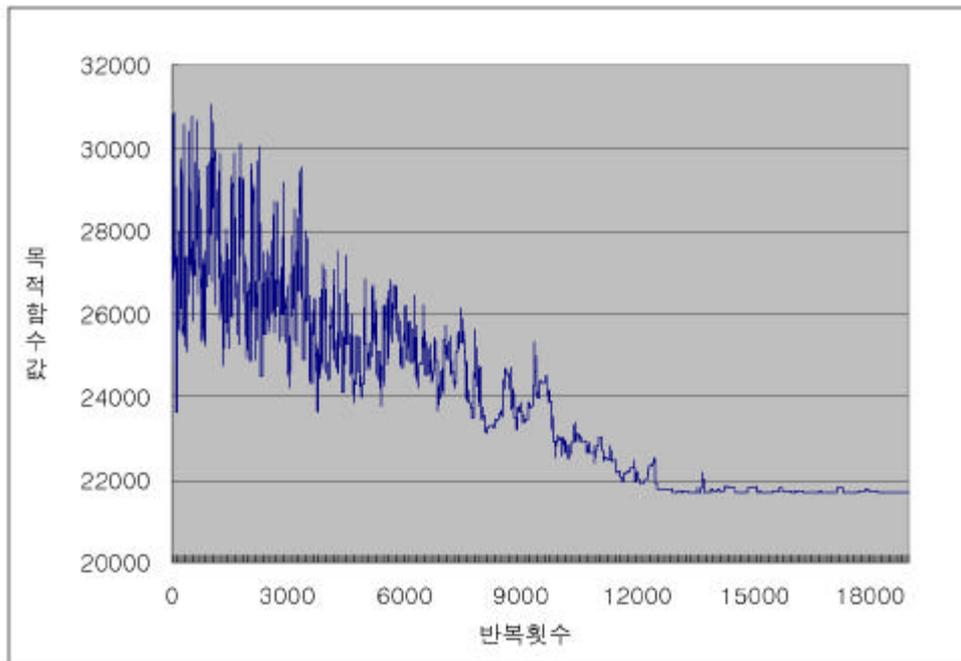


Fig. 4.10 $n=20$, Type 2 Hybrid GA SA

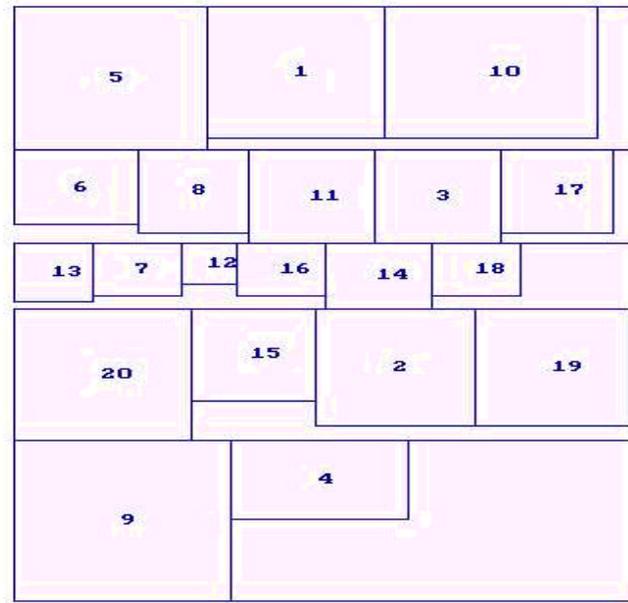


Fig. 4.11 n=20 Type 1

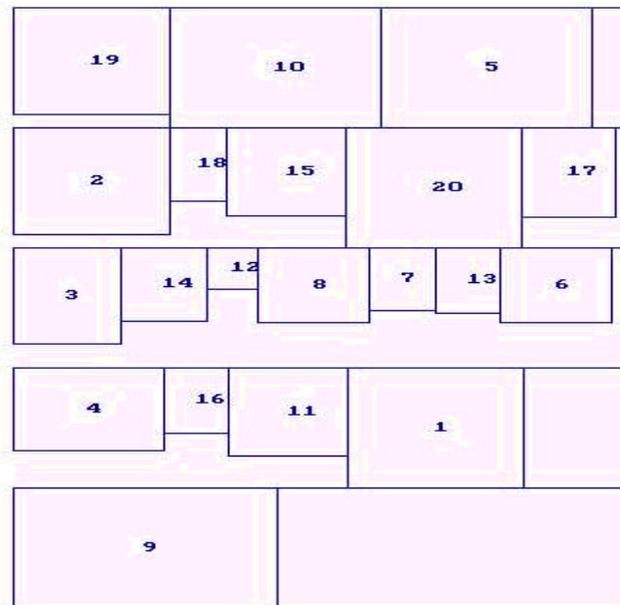


Fig. 4.12 n=20 Type 2

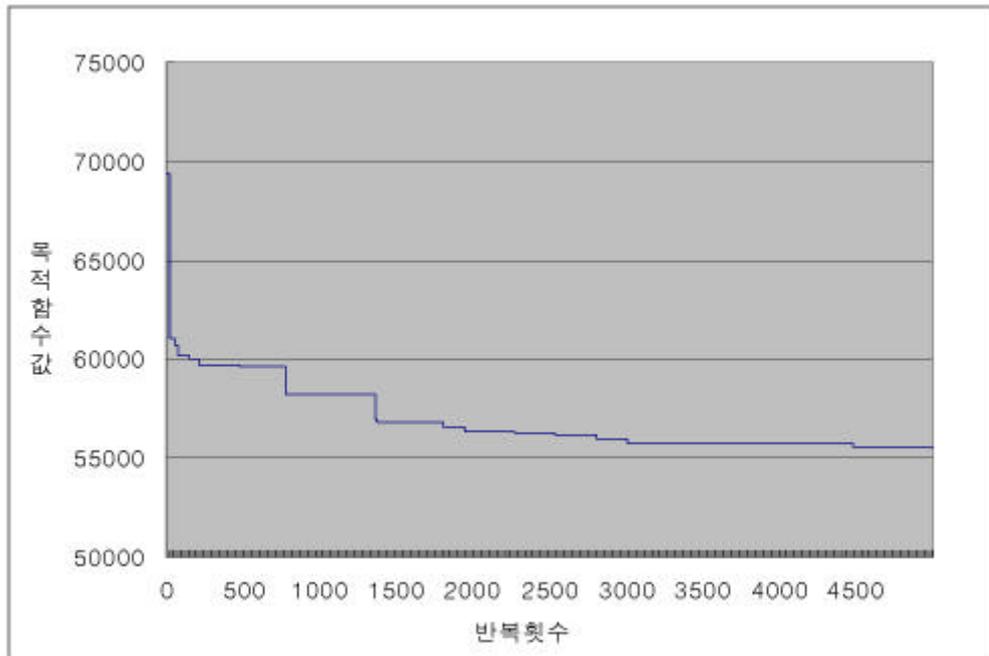


Fig. 4.13 $n=30$, Type 1 Hybrid GA GA

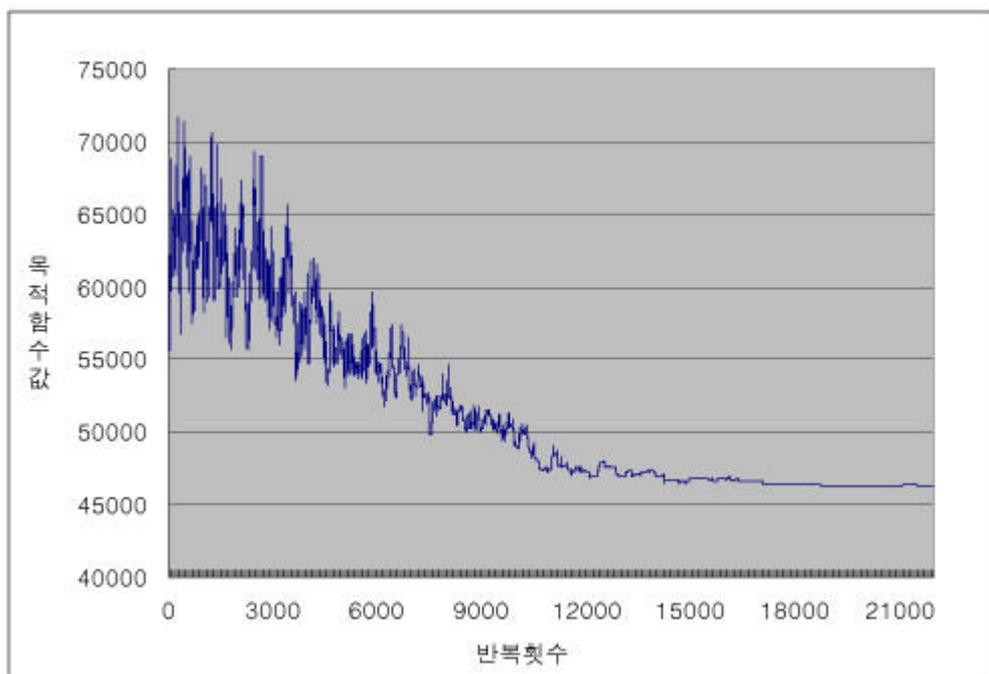


Fig. 4.14 $n=30$, Type 1 Hybrid GA SA

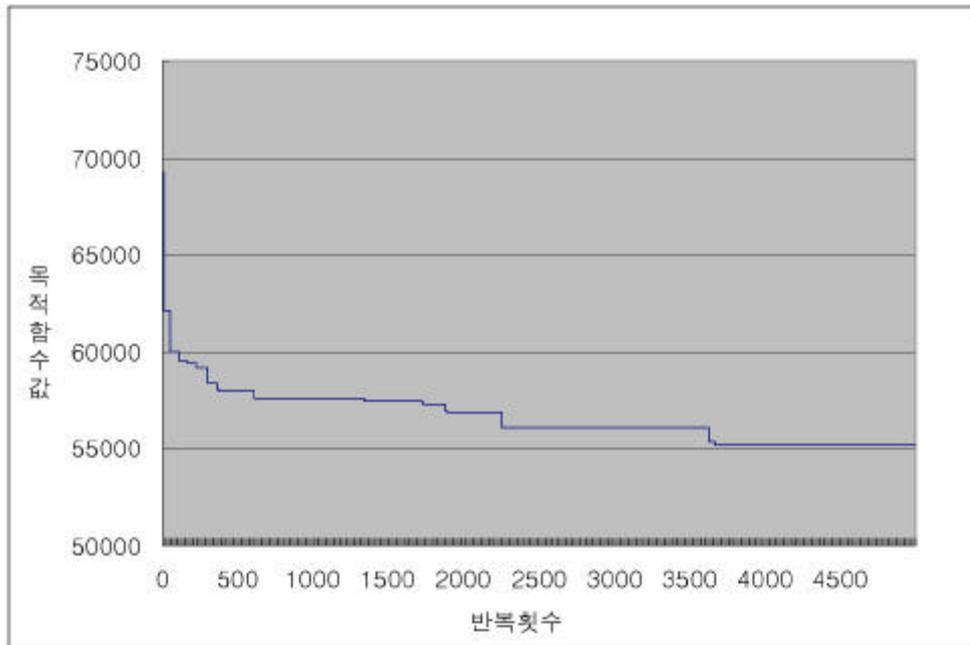


Fig. 4.15 $n=30$, Type 2 Hybrid GA GA

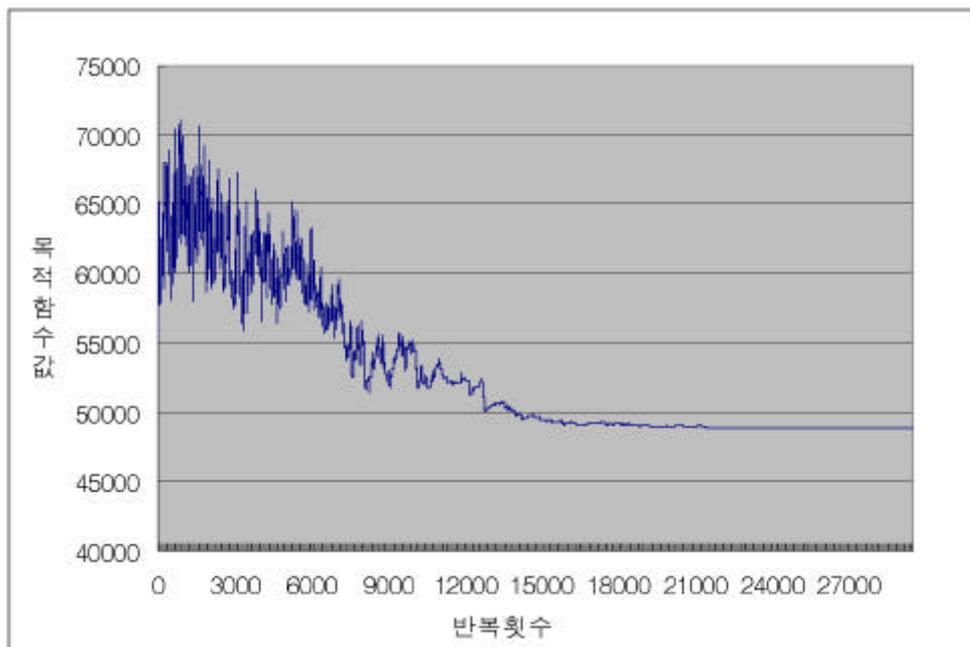


Fig. 4.16 $n=30$, Type 2 Hybrid GA SA

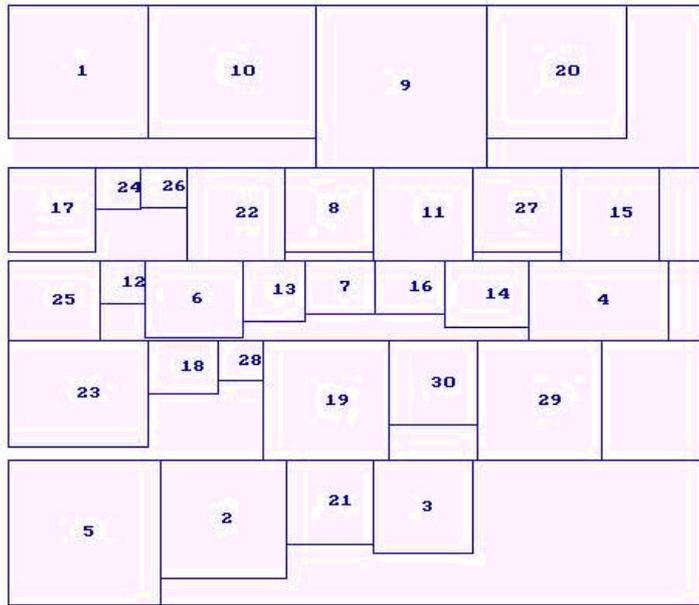


Fig. 4.17 n=30 Type 1

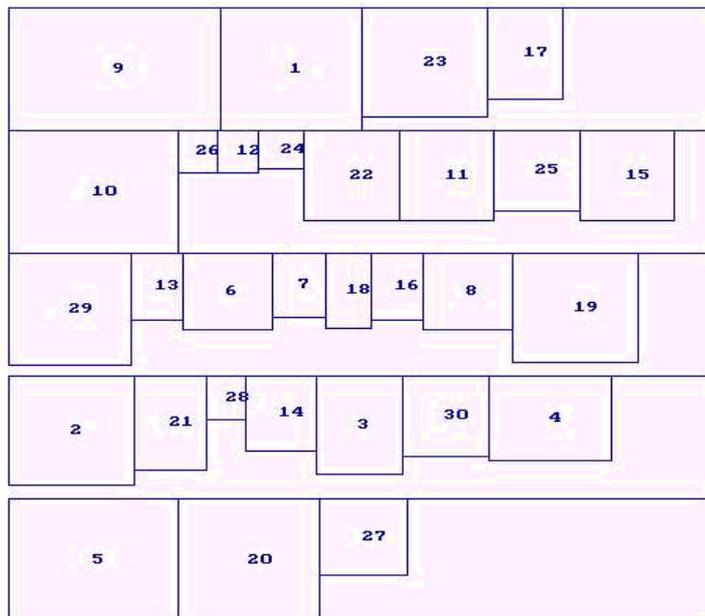


Fig. 4.18 n=30 Type 2

5

가 가
가
Hybrid GA

가 GA

가

GA가 Hill-climbing

SA
가

가

GA SA Hybrid GA

Hybrid GA

, Tam
and Park GA

17 35%
2 9%

가 , Woo
가 .

20

가

가 .

가

가

.
, Hybrid GA

(parameter)

가

- [13] , , , 1998
- [14] , , , , , 1997
- [15] Tam. K. Y., "Genetic algorithms, function optimization, and facility layout, design", *European Journal of Operations Research*, Vol.63, pp.322- 346, 1992
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1. 가 /

		가 /	가 ,	
1	100	1.0	10.00	10.00
2	80	1.0	8.94	8.94
3	50	1.3	7.07	7.07
4	60	0.8	6.00	10.00
5	120	1.0	10.95	10.95
6	40	1.0	5.71	7.00
7	20	1.4	4.00	5.00
8	40	1.0	6.32	6.32
9	150	1.1	12.24	12.24
10	120	1.5	10.00	12.00
11	50	1.1	7.07	7.07
12	10	1.2	3.16	3.16
13	20	1.5	4.47	4.47
14	30	1.25	5.00	6.00
15	50	1.1	7.07	7.07
16	20	1.5	4.00	5.00
17	40	1.4	6.32	6.32
18	20	1.9	4.00	5.00
19	80	1.0	8.94	8.94
20	100	1.15	10.00	10.00
21	40	1.5	6.32	6.32
22	50	1.1	7.07	7.07
23	80	1.0	8.00	10.00
24	10	1.0	3.16	3.16
25	40	1.1	6.00	6.66
26	10	1.2	3.00	3.33
27	40	1.0	6.32	6.32
28	10	1.3	3.00	3.33
29	80	1.05	8.94	8.94
30	40	1.1	6.32	6.32

2.

(1) 가 5

n	1	2	3	4	5
1	0	5	2	4	1
2	5	0	3	0	2
3	2	3	0	0	0
4	4	0	0	0	5
5	1	2	0	5	0

(2) 가 6

n	1	2	3	4	5	6
1	0	5	2	4	1	0
2	5	0	3	0	2	2
3	2	3	0	0	0	0
4	4	0	0	0	5	2
5	1	2	0	5	0	10
6	0	2	0	2	10	0

(3) 가 7

n	1	2	3	4	5	6	7
1	0	5	2	4	1	0	0
2	5	0	3	0	2	2	2
3	2	3	0	1	0	2	5
4	4	0	1	0	5	2	2
5	1	2	0	5	0	10	0
6	0	2	2	2	10	0	5
7	0	2	5	2	0	5	0

(4) 가 8

n	1	2	3	4	5	6	7	8
1	0	5	2	4	1	0	0	6
2	5	0	3	0	2	2	2	0
3	2	3	0	0	0	0	0	5
4	4	0	0	0	5	2	2	10
5	1	2	0	5	0	10	0	0
6	0	2	0	2	10	0	5	1
7	0	2	0	2	0	5	0	10
8	6	0	5	10	0	1	10	0

(5) 가 9

n	1	2	3	4	5	6	7	8	9
1	0	3	2	0	0	2	10	5	0
2	3	0	4	0	10	4	0	0	2
3	2	4	0	3	4	0	5	5	5
4	0	0	3	0	0	0	0	2	2
5	0	10	4	0	0	5	2	0	0
6	2	4	0	0	5	0	1	2	2
7	10	0	5	0	2	1	0	10	10
8	5	0	5	2	0	2	10	0	1
9	0	2	5	2	0	2	10	1	0

(6) 가 10

n	1	2	3	4	5	6	7	8	9	10
1	0	3	2	0	0	2	10	5	0	5
2	3	0	4	0	10	4	0	0	2	2
3	2	4	0	3	4	0	5	5	5	1
4	0	0	3	0	0	0	0	2	2	0
5	0	10	4	0	0	5	2	0	0	0
6	2	4	0	0	5	0	1	2	2	1
7	10	0	5	0	2	1	0	10	10	5
8	5	0	5	2	0	2	10	0	1	3
9	0	2	5	2	0	2	10	1	0	10
10	5	2	1	0	0	1	5	3	10	0

(7) 가 15

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	10	0	5	1	0	1	2	2	2	2	0	4	0	0
2	10	0	1	3	2	2	2	3	2	0	2	0	10	5	0
3	0	1	0	10	2	0	2	5	4	5	2	2	5	5	5
4	5	3	10	0	1	1	5	0	0	2	1	0	2	5	0
5	1	2	2	1	0	3	5	5	5	1	0	3	0	5	5
6	0	2	0	1	3	0	2	2	1	5	0	0	2	5	10
7	1	2	2	5	5	2	0	6	0	1	5	5	5	1	0
8	2	3	5	0	5	2	6	0	5	2	10	0	5	0	0
9	2	2	4	0	5	1	0	5	0	0	10	5	10	0	2
10	2	0	5	2	1	5	1	2	0	0	0	4	0	0	5
11	2	2	2	1	0	0	5	10	10	0	0	5	0	5	0
12	0	0	2	0	3	0	5	0	5	4	5	0	3	3	0
13	4	10	5	2	0	2	5	5	10	0	0	3	0	10	2
14	0	5	5	5	5	5	1	0	0	0	5	3	10	0	4
15	0	0	5	0	5	0	0	0	2	5	0	0	2	4	0

(8) 가 20

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	5	0	5	2	10	3	1	5	5	5	0	0	5	4	4	0	0	1
2	0	0	3	10	5	1	5	1	2	4	2	5	0	10	10	3	0	5	10	5
3	5	3	0	2	0	5	2	4	4	5	0	0	0	5	1	0	0	5	0	0
4	0	10	2	0	1	0	5	2	1	0	10	2	2	0	2	1	5	2	5	5
5	5	5	0	1	0	5	6	5	2	5	2	0	5	1	1	1	5	2	5	1
6	2	1	5	0	5	0	5	2	1	6	0	0	10	0	2	0	1	0	1	5
7	10	5	2	5	6	5	0	0	0	0	5	10	2	2	5	1	2	1	0	10
8	3	1	4	2	5	2	0	0	1	1	10	10	2	0	10	2	5	2	2	10
9	1	2	4	1	2	1	0	1	0	2	0	3	5	5	0	5	0	0	0	2
10	5	4	5	0	5	6	0	1	2	0	5	5	0	5	1	0	0	5	5	2
11	5	2	0	10	2	0	5	10	0	5	0	5	2	5	1	10	0	2	2	5
12	5	5	0	2	0	0	10	10	3	5	5	0	2	10	5	0	1	1	2	5
13	0	0	0	2	5	10	2	2	5	0	2	2	0	2	2	1	0	0	0	5
14	0	10	5	0	1	0	2	0	5	5	5	10	2	0	5	5	1	5	5	0
15	5	10	1	2	1	2	5	10	0	1	1	5	2	5	0	3	0	5	10	10
16	4	3	0	1	1	0	1	2	5	0	10	0	1	5	3	0	0	0	2	0
17	4	0	0	5	5	1	2	5	0	0	0	1	0	1	0	0	0	5	2	0
18	0	5	5	2	2	0	1	2	0	5	2	1	0	5	5	0	5	0	1	1
19	0	10	0	5	5	1	0	2	0	5	2	2	0	5	10	2	2	1	0	6
20	1	5	0	5	1	5	10	10	2	2	5	5	5	0	10	0	0	1	6	0

(9) 가 30

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	0	3	2	0	0	2	10	5	0	5	2	5	0	0	2	0	5	6	3	0	1	10	0	10	2	1	1	1	0	1
2	3	0	4	0	10	4	0	0	2	2	1	0	5	0	0	0	0	2	0	1	6	1	0	1	2	2	5	1	10	5
3	2	4	0	3	4	0	5	5	5	1	4	1	0	4	0	4	0	6	3	2	5	5	2	1	0	0	3	1	0	2
4	0	0	3	0	0	0	0	2	2	0	6	0	2	5	2	5	1	1	1	1	2	2	4	0	2	0	2	2	5	5
5	0	10	4	0	0	5	2	0	0	0	0	2	0	0	0	0	2	1	0	0	2	0	5	1	0	2	1	0	2	1
6	2	4	0	0	5	0	1	2	2	1	4	10	10	2	5	5	0	5	0	0	0	10	0	0	0	4	0	10	1	1
7	10	0	5	0	2	1	0	10	10	5	10	10	6	0	0	10	2	1	10	1	5	5	2	3	5	0	2	0	1	3
8	5	0	5	2	0	2	10	0	1	3	5	0	0	0	2	4	5	2	10	6	0	5	5	2	5	0	5	5	0	2
9	0	2	5	2	0	2	10	1	0	10	2	1	5	2	0	3	0	2	0	0	4	0	5	2	0	5	2	2	5	2
10	5	2	1	0	0	1	5	3	10	0	5	5	6	0	1	5	5	0	5	2	3	5	0	5	2	10	10	1	5	2
11	2	1	4	6	0	4	10	5	2	5	0	0	0	1	2	1	0	2	0	0	0	6	6	0	4	5	3	2	2	10
12	5	0	1	0	2	10	10	0	1	5	0	0	5	5	2	0	0	0	0	2	0	4	5	10	1	0	0	0	0	1
13	0	5	0	2	0	10	6	0	5	6	0	5	0	2	0	4	2	2	1	0	6	2	1	5	5	0	0	1	5	5
14	0	0	4	5	0	2	0	0	2	0	1	5	2	0	2	1	0	5	3	10	0	0	4	2	0	0	4	2	5	5
15	2	0	0	2	0	5	0	2	0	1	2	2	0	2	0	4	5	1	0	1	0	5	0	2	0	0	5	1	1	0
16	0	0	4	5	0	5	10	4	3	5	1	0	4	1	4	0	0	3	0	2	2	0	2	0	5	0	5	2	5	10
17	5	0	0	1	2	0	2	5	0	5	0	0	2	0	5	0	0	2	2	0	0	0	6	5	3	5	0	0	5	1
18	6	2	6	1	1	5	1	2	2	0	2	0	2	5	1	3	2	0	5	1	2	10	10	4	0	0	5	0	0	0
19	3	0	3	1	0	0	10	10	0	5	0	0	1	3	0	0	2	5	0	0	5	5	1	0	5	2	1	2	10	10
20	0	1	2	1	0	0	1	6	0	2	0	2	0	10	1	2	0	1	0	0	5	2	1	3	1	5	6	5	5	3
21	1	6	5	2	2	0	5	0	4	3	0	0	6	0	0	2	0	2	5	5	0	4	0	1	0	0	0	5	0	0
22	10	1	5	2	0	10	5	5	0	5	6	4	2	0	5	0	0	10	5	2	4	0	5	0	4	4	5	0	2	5
23	0	0	2	4	5	0	2	5	5	0	6	5	1	4	0	2	6	10	1	1	0	5	0	0	4	4	1	0	2	2
24	10	1	1	0	1	0	3	2	2	5	0	10	5	2	2	0	5	4	0	3	1	0	0	0	5	5	0	1	0	0
25	2	2	0	2	0	0	5	5	0	2	4	1	5	0	0	5	3	0	5	1	0	4	4	5	0	1	0	10	1	0
26	1	2	0	0	2	4	0	0	5	10	5	0	0	0	0	0	5	0	2	5	0	4	4	5	1	0	0	0	0	0
27	1	5	3	2	1	0	2	5	2	10	3	0	0	4	5	5	0	5	1	6	0	5	1	0	0	0	0	0	0	10
28	1	1	1	2	0	10	0	5	2	1	2	0	1	2	1	2	0	0	2	5	5	0	0	1	10	0	0	0	2	2
29	0	10	0	5	2	1	1	0	5	5	2	0	5	5	1	5	5	0	10	5	0	2	2	0	1	0	0	2	0	2
30	1	5	2	5	1	1	3	2	2	2	10	1	5	5	0	10	1	0	10	3	0	5	2	0	0	0	10	2	2	0

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