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Optimization Method for Parameter Estimation of Cap Model

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ABSTRACT

In this paper, the method of parameter estimation of a mathematical constitutive model known as the smooth elasto-plastic cap model is studied. To predict the response of the real soil using this model, the eight parameters describing the constitutive equations have to be determined. First, an experimental data is obtained from simple laboratory experiments such as one dimensional confined compression test in a consolidometer with the Ottawa sand for the reference value. Then, the numerical experiment is performed in the cap model with initial guessed parameters. The optimization method is utilized to fit the model response to experimental data by minimizing the error between the two responses.

Key words: Parameter Estimation, Cap models, Elasto-Plasticity, Optimization

1. Introduction

From a theoretical point of view, the elasto-plastic cap model (Seo, 2001) is particularly appropriate to describe the soil behavior, because it allows the control of dilatancy by means of moving hardening cap. Once a mathematical constitutive law consistent with the physical behaviors is derived, it is necessary to identify and choose all significant parameters that are needed to define it. Several methods for determining the unknown parameters have been presented in the literature. The standard curve fitting method (Desai and Siriwardane, 1984; Zaman et al., 1982) was used based on physical insight into the experimental data. Although this procedure provides a parameter fitting inspired by construction of the numerical model, it has some drawbacks: (1) a large amount of conventional experimental data is required; and (2) it is not possible to use some existing non-conventional experimental data. Gauss-Newton method (Matsui et al., 1994) and Marquardt-Lenvenberg method (Simo et al., 1998) were also reported based on optimization techniques. Gauss-Newton is simple and uses a limited amount of test data, however there is a major drawback. The coefficient matrix of simultaneous equations can be singular or nearly singular which leads to numerical instabilities. In this case, if the objective function becomes completely insensitive to any of the design variables during the optimization iteration, and then the matrix will be rank deficient. In this study, alternative constrained optimization procedure which are using the existing optimization code such as IDESIGN (Arora, 1997) embedded SQP algorithm. In the simulation reported herein, a test of simple

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laboratory experimental data using the Ottawa sand for the reference value, are used to define the eight material parameters ($\alpha, \theta, W, D, \chi, H, \lambda$ and μ) that make up the cap model. The detailed parameter estimation procedure will be explained as follows.

2. Description of the cap model

In this section, the basic constitutive equations of a smooth three surface cap model (Seo, 2001) are first summarized. Utilizing the assumption of small deformation, the strain tensor admits the additive elasto-plastic decomposition;

$$\epsilon = \epsilon^e + \epsilon^p \tag{1}$$

where ϵ, ϵ^e and ϵ^p are the total, elastic, and plastic strain tensors, respectively. The elastic response of the material is assumed to be characterized by a constant isotropic tensor $C = K 1 \otimes 1 + 2\mu I_{dev}$ such that the incremental stress response of the material is given by

$$\dot{\sigma} = C : (\dot{\epsilon} - \dot{\epsilon}^p) \tag{2}$$

where K is the bulk modulus of the soil and μ is the shear modulus. In stress space, the elastic domain is bounded by three distinct yield surfaces which are functions of the two invariants $I_1 = tr(\sigma)$ and $\|s\|$, where s is the deviatoric part of the stress tensor $\sigma (i.e.\ s = I_{dev} : \sigma)$. The three surfaces comprising the yield surface intersect in a smooth manner as shown in Figure 1.

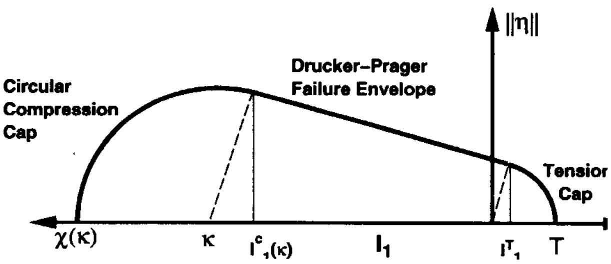

Fig. 1 Smooth, three-surface, two invariant yield function for cap

The form of the yield function, $f_m(\sigma, \kappa)\ (m = 1, 2, 3)$ are specified in terms of functions F_e, F_c and F_t which are respectively called the Drucker-Prager envelope function, the compression cap function, and the tension cap function. The mathematical forms are

$$f_1(\sigma, q) = |\eta|^2 - F_e(I_1) \leq 0 \tag{3}$$

$$f_2(\sigma, q, \kappa) = \|\eta\|^2 - F_c(I_1, \kappa) \leq 0 \tag{4}$$

$$f_3(\sigma, q) = \|\eta\|^2 - F_t(I_1) \leq 0 \tag{5}$$

where: $\eta = s - q$ and $\|\eta\|^2 = [\eta : \eta]^{1/2}$. As is customary s denotes the deviatoric stress, and q denotes a purely deviatoric back stress associated with kinematic hardening. The specific forms of F_e, F_c and F_t are defined here as

$$F_e(I_1) = \alpha - \theta I_1 \qquad I_1^C(\kappa) \leq I_1 \leq I_1^T \tag{6}$$

$$F_c(I_1, \kappa) = R^2(\kappa) - (I_1 - \kappa)^2 \qquad I_1 < I_1^C(\kappa) \tag{7}$$

$$F_t(I_1) = R_T^2 - I_1^2 \qquad I_1 > I_1^T \tag{8}$$

In the preceding expressions, α and θ are basic material constants. Approximate translation from Mohr-Coulomb parameters to Drucker-Prager parameters have been provided as for example in Chen and Saleeb(1982) as

$$\alpha = \frac{\sqrt{2}c}{(1 + 4/3\, tan^2\phi)^{1/2}} \quad \text{and} \quad \theta = \frac{\sqrt{2}\tan\phi}{3(1 + 4/3\, tan^2\phi)^{1/2}} \tag{9}$$

Whereas the entities I_1^T and $I_1^C(\kappa)$ denotes, respectively, a fixed delimiting point between the Drucker-Prager envelope and the tension cap, and the Drucker-Prager envelope and the compression cap. Specific expression for these points are

$$I_1^T = \alpha \cos(\phi) \sin(\phi) \tag{10}$$

$$I_1^C = \kappa + R(\kappa) \sin(\phi) \tag{11}$$

where $\phi = tan^{-1}(\theta)$. As the compression cap surface translates along the I_1 axis, the cap surface radius $R(\kappa)$ changes as a function of the cap parameter κ as follows

$$R(\kappa) = -\kappa \sin(\phi) + \alpha \cos(\phi) \tag{12}$$

The tension cap surface is circular. The center of tension cap resides at $I_1 = 0$, and the radius of the surface is a constant R_T which is expressed as

$$R_T = \alpha \cos(\phi) \tag{13}$$

The hardening law for this model derives from the fact that the volumetric crush curve (plastic volumetric strain ϵ_v^p versus I_1) is assumed to be an exponential of the form

$$\epsilon_v^p = - W[1 - exp\{DX(\kappa)\}] \tag{14}$$

Differentiating equation with respect to κ allows us to obtain a variable tangent hardening modulus $h'(\kappa)$ for κ as follows

$$h'(\kappa) = \frac{d\epsilon_v^p}{d\kappa} = \frac{exp(-DX)}{WDX'} \tag{15}$$

where $X' = 1 - RF_e(\kappa)$; W represents the maximum possible plastic volumetric strain for the medium, with the reference state being the material's virgin unloaded state; and D^{-1} denotes the absolute value of I_1 at which $e^{-1} \cdot 100\%$ of the medium's original crushable porosity remains. This nonlinear hardening modulus $h'(\kappa)$ is used to provide a nonlinear incremental hardening law governing movement of the cap parameter

$$\dot{\kappa} = h'(\kappa)\, tr(\dot{\epsilon}^p) \tag{16}$$

A purely deviatoric linear kinematic hardening law is employed with this model, the rate form of which is

$$\dot{q} = HI_{dev} \cdot \dot{\epsilon}^p \tag{16}$$

where H is a constant plastic hardening modulus.

The flow rule for this model is associated, and since multiple surfaces are potentially active at any given instant, it takes Koiter's generalized form

$$\dot{\epsilon}^p = \sum_m \dot{\gamma}^m \frac{\partial f_m}{\partial \sigma} \tag{17}$$

3. Sequential quadratic programming

This method uses the Taylor series expansion to liberalize a nonlinear optimization problem and liberalized subprogram in transformed to a quadratic program. While this method uses the basic idea as the Gauss-Newton method, it treats the optimal fitting process as a least square constrained optimization problem. The constrains imposed on the optimization problem are in the sense of physically meaningful bounds. The formal statement of the optimization problem is

$$MIN\ J(b) \equiv \sum_{j=1}^{N} (u_j - z_j(b))^2$$

$$\text{subjected to } a \le b \le c \tag{18}$$

where N is number of observation; u_j is observed response from the laboratory experiments; z_j is response from constitutive model; j is j^{th} data point; b is design parameter vector; a and c are lower and upper bounds of design variables. There exists a wide variety of algorithms to solve the above constrained optimization problems. To avoid implementing a such algorithms, the existing optimization code IDESIGN (Arora, 1997) was used here due to its robustness and generality. The sequential quadratic programming algorithms can be found in Arora (1989).

Since the magnitude of cap model parameters are vastly different in size, it is important to normalize the design variables for better performance of the optimization process. The normalized design variables b_i are defined as

$$b_i = \frac{x_i}{k_i} \tag{19}$$

where x_i are the i^{th} original design variables and k_i are their normalization factors. Thus, using appropriate k_i, the design variables can be force to vary approximately between -1 to +1. The derivatives of a objective function $J(x_i)$ with respect to normalized variables b_i are given as

$$\frac{\partial J}{\partial b_i} = \frac{\partial J}{\partial x_i}\frac{\partial x_i}{\partial b_i} = \frac{\partial J}{\partial x_i}k_i \tag{20}$$

4. Parameter estimation and results

In order to access the capability of smooth cap model in predicting response behavior of a real material, model parameters needed to be estimated from laboratory experimental data. In this section parameter estimation procedure for one dimensional confined compression test and with Ottawa sand is presented, followed by the numerical simulation result. Satisfactory agreements are achieved between experimental data and numerical model responses.

4.1. One dimensional compression test

For the uniaxial strain test conducted in the laboratory, the dry sand sample was loaded axially under stress controlled mode. The schematic diagram for the test is shown in Figure 2.

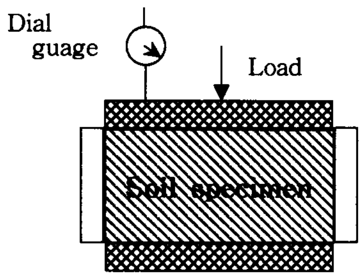

Fig. 2. Diagram of experimental 1-D compression test

A general procedure for the estimation of material parameters used by the constitutive model follows; 1) initial normalized design variables b_i are assumed within physically reasonable bounds; 2) the design variables were unscaled ($x_i = k_i \cdot b_i$) to calculate the response from the constitutive model; 3) the objective function and the gradient were computed. The estimation algorithm is shown in Figure 3. A comparison between experimental and predicted curve is shown in Figure 4. The values of the material parameters obtained in the estimation process and the boundary values used are summarized in Table 1.

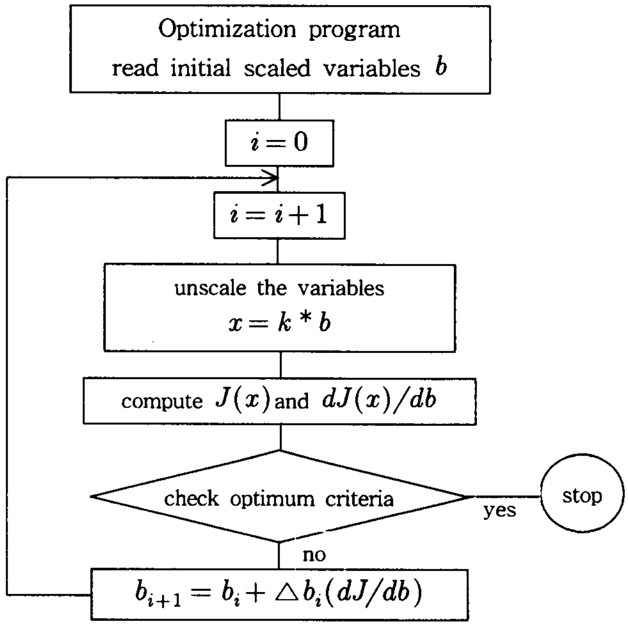

Fig. 3. Optimization algorithm for 1-D compression test

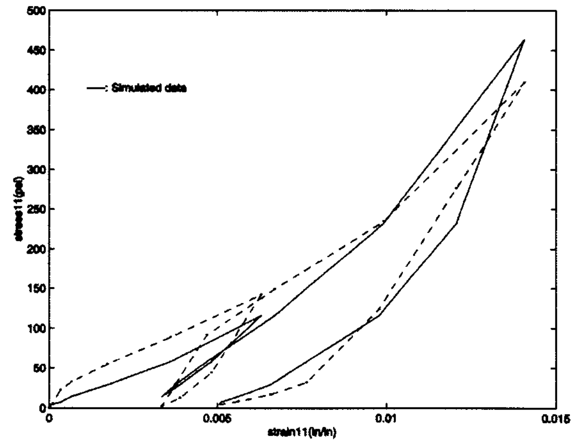

Fig. 4 Simulation result

Table 1 Optimal value of parameters

Material Parameter (Upper and lower bound)	Uniaxial strain test (Optimal value)
$0 \le \lambda \le 10^9\,(kN/m^2)$	2.3719×10^7
$0 \le \mu \le 10^9\,(kN/m^2)$	1.1101×10^7
$0 \le \alpha \le 10^5\,(N/m^2)$	2.9717×10^2
$0 \le \theta \le 0.5$	0.2727
$0 \le W \le 0.1$	0.0157
$0 \le D \le 10^{-3}\,(kN/m^2)^{-1}$	2.994×10^{-4}
$-10^5 \le \kappa \le 0\,(N/m^2)$	-100

5. Conclusion

The parameter estimation procedures for the soil model have been presented to identify the eight parameters of the smooth cap model using Ottawa sand. An experimental data is first obtained from uniaxial strain test which is considered to be representative of material response. Then, the numerical simulations with initially guessed values are performed until the model responses are matched with the experimental test results.

References

1. Arora, J. S, 1989, Introduction to Optimal Design, McGraw-Hill Series.
2. Arora, J. S., Lin, T., Elwakeil, O. and Haung, M. 1997, IDESIGN users' manual version 4.2, Optimal Design Laboratory, The Univ. of Iowa, Iowa City
3. Chen, W.-F. and Saleeb, A. F. 1982, Constitutive Equations for Engineering Material, Vol. 1: Elasticity and Modeling, John-Wiley.
4. Desai, C. S. and Siriwardance, H. I., 1984, Constitutive laws for engineering materials, Prentice Hall, Englewood-Cliffs, NJ..
5. Matsui, K., Nishida, N, Dobashi, Y. and Ushioda, K. 1994, An inverse method for estimation of termal properties of mass concrete, Transations of Japan Concrete Institute, 16, 131-138.
6. Seo, Y-K, 2001, A smooth Elasto-Plastic Cap Model(I): Rate Formulation, Yield Surface Determination, J. of Korean Geotech. Society, .17(3).
7. Simo, J. C., Ju, J,-W., Pister, K. S. and Taylor, R. L. 1988, Assessment of cap model: Consistent return algorithms and rate-dependent extension, J. Eng. Mech. 114, 191-218.
8. Zaman, M. M., Desai, C. S. and Frauque, M. O., 1982, An algorithm for determining parameters for cap model from raw laboratory test data, Proc. 4th Int. Conf. Numer. Meth. Geomech. Edmonton, Canada.