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On the Fuzzy θ-Continuous Mappings

Park, Jin Han, Lee, Bu young**, and Lee, Jong Hoo****

Abstract: The concepts of fuzzy almost continuous, fuzzy θ-continuous and fuzzy weakly continuous have been introduced in [9,14]. The aim of this paper is mainly to study and to find the mutual interrelations among these concepts.

I. Introduction

The concepts of fuzzy almost continuity and fuzzy weakly continuity, defined by Azad [1], were introduced by Yalvac [14] using the concept of quasi-coincidence. Recently, Mukherjee and Sinha [8,9] also defined and studied certain types of near-fuzzy continuous mappings between fuzzy topological spaces, some of which were independent of and the restrictly weaker than fuzzy continuous mappings.

The notions of fuzzy δ-closure and fuzzy δ-closure of a fuzzy set in fuzzy topological space were introduced by Ganguly and Saha [4] and Mukherjee and Sinha [9], respectively. These concepts were very interesting in the study of those some near-fuzzy continuous mappings between fuzzy topological spaces.

In this paper, we show that fuzzy almost continuity implies fuzzy θ-continuity and to give some sufficient conditions for fuzzy θ-continuous (fuzzy weakly continuous) mapping to be fuzzy almost continuous. And we study some properties of fuzzy θ-continuous mappings on fuzzy topological spaces.

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*,** Department of Mathematics Dong-A University

*** Department of Applied Mathematics Korea Maritime University Pusan 606, Korea

2. Preliminaries

Fuzzy sets of a nonempty set X will be denoted by the capital letter as A, B, C, etc. The value of a fuzzy set A at the element x of X will be denoted by $A(x)$, and fuzzy points will be denoted by p and r. And $p \in A$ means that fuzzy point p takes its non-zero value in [0,1] at the support x_p and $p(x_p) \leq A(x_p)$ (see in [6]). For definitions and results not explained in this paper, the reader were referred to the papers [1,2,13,14 and 15] assuming them to be well known. The words "fuzzy", "neighborhood" and "fuzzy topological space" will be abbreviated as "f", "nbd" and "fts", respectively.

Definition 2.1 [6]. Let A and B be f.sets of X and let p be f.point in X. p is said to be quasi-coincident with A, denoted by pqA, if $p(x_p) + A(x_p) > 1$ or $p(x_p) > A'(x_p)$. A is said to be quasi-coincident with B, denoted by AqB, if there exists $x \in X$ such that $A(x)+B(x) > 1$.

Theorem 2.1 [6]. Let A and B be f.sets of X. $A \leq B$ if and only if A and B' are not quasi-coincident denoted by A/B'.

Theorem 2.2 [2,14]. Let $f:X \longrightarrow Y$ be a mapping and let A and B be f.set of X and Y, respectively. The following statements are true:

(a) $f(A)' \leq f(A')$, $f^{-1}(B') = f^{-1}(B)'$.

(b) $A \leq f^{-1}(f(A))$, $f(f^{-1}(B)) \leq B$.

(c) If f is one-to-one, then $f^{-1}(f(A)) = A$.

(d) If f is onto, then $f(f^{-1}(B)) = B$.

(e) If f is one-to-one and onto, then $f(A)' = f(A')$.

Let f be a mapping from X to Y. Clearly for every f.point p in X, $f(p)$ is a f.point in Y, and if supp$p = x_p$ then supp$(f(p)) = f(x_p)$, $f(p)(f(x_p)) = p(x_p)$. If p is a f.point in Y then $f^{-1}(p)$ need not be a f.point in X. If f is one-to-one and p is a f.point in $f(X)$ then $f^{-1}(p)$ will be f.point in X In this case if supp$p=y_P$ then supp$f^{-1}(p)=f^{-1}(y_P)$ and $f^{-1}(p)(f^{-1}(y_P))=p(y_P)$[14].

Theorem 2.3 [14]. Let $f:X\longrightarrow Y$ be a mapping and let p be a f.point in X.

(a) If $B\leq Y$ and $f(p)qB$, then $pqf^{-1}(B)$.

(b) If $A\leq X$ and pqA, then $f(p)qf(A)$.

Definition 2.2 [6]. Let X be fts and A be f.set of X. A is q-nbd of a f.point p if there exists a f.open set B such that $pqB\leq A$.

Theorem 2.4 [6]. Let A be f.set of X and p be a f.point. $p\in\overline{A}$ if and only if for each q-nbd B of p, BqA.

3. Fuzzy θ-continuity

Let f be a mapping from a fts X to another fts Y.

Definition 3.1. f is said to be

(a) f.θ-continuous if for each f.point p in X and each f.open q-nbd V of $f(p)$, there is a f.open q-nbd U of p such that $f(\overline{U})\leq\overline{V}$[9],

(b) f.almost continuous if for each f.point p in X and each for f.open q-nbd V of $f(p)$, there is a f.open q-nbd U of p such that $f(U)\leq(\overline{V})^{o}$[14],

(c) f.weakly continuous if for each f.point p in X and each f.open q-nbd V of $f(p)$, there is a f.open q-nbd U of p such that $f(U)\leq\overline{V}$[14].

Theorem 3.1. Every f.almost continuous mapping is f.θ-continuous.

Proof. Let $f:X\longrightarrow Y$ be f.almost continuous. Let p be f.point in X and let V be f.open q-nbd of $f(p)$. Then by f.almost continuity of f, there is a f.open q-nbd U of p such that $f(U)\leq(\overline{V})^{o}$. We show that $f(\overline{U})\leq\overline{V}$ and then complete the proof. Let there exists a f.point r in X such that $r\in\overline{U}$ and $f(r)\notin\overline{V}$. Since

$f(r)(f(x_r))\nleq \bar{V}(f(x_r))$, $f(r)q(\bar{V})'$. Then $(\bar{V})'$ is f.open q-nbd of $f(r)$. Since f is f.almost continuous, there exists f.open q-nbd W of r such that $f(W)\leq \overline{((\bar{V})')^{\circ}}=(\bar{V})'$. But since $r\in\bar{U}$, we have WqU. Hence $f(U)q(\bar{V})'$. This is contrary to the fact that $f(U)\leq(\bar{V})^{\circ}\leq\bar{V}$.

Clearly, f.θ-continous mapping is f.weakly continuous. And f.almost continuous mapping is f.θ-continuous. But that the converse need not be true is shown the following examples.

Example 3.1. Let $X=[0,1]$, $\tau_1=\{X,\phi,A\}$ and $\tau_2=\{X,\phi,B\}$, where

$$A(x)=\begin{cases}\frac{1}{4}, & x=0\\ 0, & x\neq 0,\end{cases}\quad B(x)=\begin{cases}\frac{1}{5}, & x=0\\ 0, & x\neq 0.\end{cases}$$

Let $f:(X,\tau_1)\longrightarrow(X,\tau_2)$ be the identity mapping. Then f is f.θ-continuous but not f.almost continuous.

Let p be a f.point in (X,τ_1) and V be any f.open q-nbd of $f(p)$ in (X,τ_2). If $V=B$, then we choose $U=A$ and then U is a f.open q-nbd of p such that $f(\bar{U})=A'$ Again, $\bar{V}=\bar{B}=B'$. Hence $f(\bar{U})\leq\bar{V}$. In case $V=X$, the conclusion is obvious. Hence f is f.θ-continuous.

Let us consider the f.point $p=(0,\frac{5}{6})$. Now B is a f.open q-nbd of $f(p)$ in (X,τ_2), and A and X are f.open q-nbd of p in (X,τ_1). But $f(A)\nleq(\bar{B})^{\circ}$ and $f(X)\nleq(\bar{B})^{\circ}$. Thus f is f.almost continuous.

Exalpe 3.2. Let $X=[0,1]$, $\tau_1=\{X,\phi,A,B\}$ and $\tau_2=\{X,\phi,C\}$, where

$$A(x)=\begin{cases}\frac{1}{5}, & x=0\\ 0, & x\neq 0,\end{cases}\qquad B(x)=\begin{cases}\frac{11}{20} & x=0\\ 0, & x\neq 0.\end{cases}\qquad C(x)=\begin{cases}\frac{1}{4}, & x=0\\ 0, & x\neq 0\end{cases}$$

Let $f:(X,\tau_1)\longrightarrow(X,\tau_2)$ be the identity mapping. Then f is f.weakly continuous but not f.θ-continuous.

Let p be a f.point in X. If $x_p \neq 0$, then $V=X$ is only f.open q-nbd of $f(p)$, and then $U=X$ is a f.open q-nbd of p such that $f(U)=V$. Suppose $x_p=0$, and V is a f.open q-nbd of $f(p)$. If $V=X$, the case becomes trivial. So let $V=C$. Then $p(x_p) > \frac{3}{4}$ so that B is a f.open q-nbd of p such that $f(B)=B\leq\bar{C}=C'$. Thus f is f.weakly continuous.

Now consider the f.point $p=(0,\frac{31}{40})$ in X. Then C is a f.open q-nbd of $f(p)$. Let U be any f.open q-nbd of p. Then $U=B$ or X, and $f(\bar{U})=A'$ or X (according as $U=B$ or X) $\not\leq\bar{C}=C'$. Hence f is not f.θ-continuous.

Definition 3.2 [4]. A f.point p is said to be a f.δ-cluster point of a f.set A if and only if every f.regularly open q-nbd U of p is q-coincident with A. The set of all f.δ-cluster points of A is called the f.δ-closure of A and is denoted by $[A]_\delta$. A f.set A is f.δ-closed if and only if $A=[A]_\delta$ and the complement of a f.δ-closed set is called f.δ-open.

Definition 3.3 [9]. A f.point p is said to be a f.θ-cluster point of a f.set A if and only if for every f.open q-nbd U of p, $\bar{U}$ is q-coincident with A. The set of all f.θ-cluster point of A is called the f.θ-closure of A and is denoted by $[A]_\theta$. A f.set A is f.θ-closed if and only if $A=[A]_\theta$ and the complement of a f.θ-closed set is f.θ-open.

It is easy to see that $\bar{A}\leq[A]_\delta\leq[A]_\theta$, for any f.set A in a fts X. And for a f.open set A in a fts X, $\bar{A}=[A]_\delta=[A]_\theta$[9].

Lemma 3.1. For a f.semi-open set A in a fts X, $\bar{A}=[A]_\delta$.

Proof. It is sufficient to show that $[A]_\delta\leq\bar{A}$. Let $p\in[A]_\delta$ such that $p\in\bar{A}$.

Then there exists a f.open q-nbd V of ρ such that $V\not{q}A$. Since A is f.semi-open, there exists a f.open set G such that $G\leq A\leq\bar{G}$. Then $V\leq A'\leq G'\Rightarrow\bar{V}\leq\bar{G'}=G'\Rightarrow(\bar{V})^{o}\leq(G')^{o}\leq G'\Rightarrow(\bar{V})^{o}\not{q}G\Rightarrow G\leq((\bar{V})^{o})'$. Then $A\leq\overline{((\bar{V})^{o})'}=((\bar{V})^{o})'\Rightarrow(\bar{V})^{o}\not{q}A\Rightarrow\rho\notin[A]^{-}$. This is contrary to the fact that $\rho\in[A]^{-}$.

Definition 3.4 [9]. A f.set A in fts X is said to be a f.δ-nbd(f.θ-nbd) of a f.point ρ if and only if there exists a f.requlary open q-nbd(f.closed q-nbd) V of ρ such that VqA'.

Theorem 3.2. If f:X$\longrightarrow$Y is a mapping, then the following are equivalent:

(a) f is f.almost continuous.

(b) $f(\bar{U})\leq[f(U)]^{-}$ for every f.set U in X.

(c) The inverse image of every f.δ-closed set in Y is f.closed set in X.

(d) The inverse image of every f.δ-open set in Y is f.closed set in X.

(e) For each f.point ρ in X and each f.δ-nbd N of $f(\rho)$, $f^{-1}(N)$ is a q-nbd of ρ.

Proof. (a)$\Rightarrow$(b): Let $\rho\in\bar{U}$ and let V be a f.open q-nbd of $f(\rho)$. Then there exists a f.open q-nbd W of ρ such that $f(W)\leq(\bar{V})^{o}$. Now, $\rho\in\bar{U}\Rightarrow WqU\Rightarrow f(W)qf(U)\Rightarrow(\bar{V})^{o}qf(U)\Rightarrow f(\rho)\in[f(U)]^{-}\Rightarrow\rho\in f^{-1}([f(\bar{U})]^{-})$. Thus $U\leq f^{-1}([f(\bar{U})]^{-})$ so that $f(U)\leq[f(U)]^{-}$.

(b)$\Rightarrow$(c): Let K be f.δ-closed set in Y. Then $[K]^{-}=K$ and hence by (a), $f\,\overline{(f^{-1}(K))}\leq[f(f^{-1}(K))]^{-}\leq[K]^{-}=K$ so that $\overline{f^{-1}(K)}\leq f^{-1}(K)$. Thus $f^{-1}(K)$ is f.closed set.

(c)$\Rightarrow$(d): Let K be f.δ-open in Y. Then K' is f.δ-closed and by (c), $f^{-1}(K')$ is f.closed. Since $f^{-1}(K')=(f^{-1}(K))'$, $f^{-1}(K)$ is f.open.

(d)$\Rightarrow$(a): Let ρ be a f.point in X and V any f.open q-nbd of $f(\rho)$. Then $(\bar{V})^{o}$ is a f.regularly open q-nbd of $f(\rho)$. Since f.regularly open sets are f.δ-open, $(\bar{V})^{o}$ is f.δ-open. By (d), $f^{-1}((\bar{V})^{o})$ is a f.open set in X and $\rho\notin(f^{-1}((\bar{V})^{o}))'$. Putting $B=(f^{-1}((\bar{V})^{o}))'$, since B is a f.closed set, there exists a f.open q-nbd U of p such that $U\not{q}B$. Then $pqU \leq B' = f^{-1}((\bar{V})^o)$ which proves that $f(U) \leq (\bar{V})^o$. Next we show that $p \notin (f^{-1}((\bar{V})^o))'$. If $p \in (f^{-1}((\bar{V})^o))'$, then

$$f(p)(f(x_p)) = p(x_p) \leq (f^{-1}((\bar{V})^o))'(x_p) = ((\bar{V})^o)'(f(x_p)). \qquad (1)$$

Now since V is f.open, $V \leq (\bar{V})^o$ so that $V'(f(x_p)) \geq ((\bar{V})^o)'(f(x_p))$. Then since V is a q-nbd of $f(p)$, we have $f(p)(f(x_p)) + V(f(x_p)) > 1$ which implies

$$f(p)(f(x_p)) > V'(f(x_p)) \geq ((\bar{V})^o)'(f(x_p)). \qquad (2)$$

Clearly, (1) and (2) are incompatible.

(a)⇒(e): Let p be a f.point in X and N any f.δ-nbd of $f(p)$. Then there exists a f.open q-nbd V of $f(p)$ such that $(\bar{V})^o \not{q} N'$. Since f is f.almost continuous, there exists a f.open q-nbd U of p such that $f(U) \leq (\bar{V})^o \leq N$, so that $U \leq f^{-1}(N)$ and hence $f^{-1}(N)$ is a q-nbd of p.

(e)⇒(a): Let p be a f.point in X and V any f.open q-nbd of $f(p)$ in Y. Then $(\bar{V})^o$ is a f.δ-nbd of $f(p)$. By (e), $f^{-1}((\bar{V})^o)$ is a q-nbd of p. Hence there exists a f.open set U such that $pqU \leq f^{-1}((\bar{V})^o)$ so that $f(U) \leq (\bar{V})^o$. Thus f is f.almost continuous.

Theorem 3.3. Let X, Y and Z be fts's such that Y is product related to Z. Let $f_1 : X \longrightarrow Y$ and $f_2 : X \longrightarrow Z$ be any mappings. Then $f : X \longrightarrow Y \times Z$, defined by $f(x) = (f_1(x), f_2(x))$, for all $x \in X$, is f.θ-continuous if and only if f_1 and f_2 are so.

Proof. Let p be f.point of X and U_1, U_2 be f.open q-nbd of $f_1(p)$ and $f_2(p)$ in Y and Z, respectively. Then $U_1 \times U_2$ is clearly a f.open q-nbd of $f(p)$. Since f is f.θ-continuous, there exists a f.open q-nbd V of p in X such that $f(V) \leq \overline{U_1 \times U_2} = \overline{U_1} \times \overline{U_2}$. By Lemma 2.9 (a) in [7], then we have $f_1(\bar{V}) \leq \overline{U_1}$ and $f_2(\bar{V}) \leq \overline{U_2}$, so that f_1 and f_2 are f.θ-continuous.

Conversely, let p be f.point of X and W be any f.open q-nbd of $f(p)$ in X×Z. Then by Lemma 2.9 (b) in [7], there exist f.open q-nbds U_1 of $f_1(p)$ and U_2 of $f_2(p)$ such that $f(p)qU_1 \times U_2 \leq W$. Since f_1 and f_2 are f.θ-continuous, there exist f.open q-nbds V_1 and V_2 of p in X such that $f_1(\overline{V_1}) \leq \overline{U_1}$ and $f_2(\overline{V_2}) \leq \overline{U_2}$, so that $f_1(\overline{V_1}) \times f_2(\overline{V_2}) \leq \overline{U_1} \times \overline{U_2}$. Now by hypothesis, putting $V = V_1 \cap V_2$, V is obviously a f.open q-nbd of p and $f(\overline{V}) \leq f_1(\overline{V_1}) \times f_2(\overline{V_2}) \leq \overline{U_1} \times \overline{U_2} \leq \overline{U_1 \times U_2} \leq \overline{W}$. Hence f is f.θ-continuous.

Theorem 3.4. Let X and Y be fts's such that X is product related to Y. Let $f: X \longrightarrow Y$ be a mapping and $g: X \longrightarrow X \times Y$, given by $g(x) = (x, f(x))$ for each $X \in X$, be the graph mapping. Then g is f.θ-continuous if and only if f is f.θ-continuous.

Proof. Let p be a f.point in X and V be any f.open q-nbd of $f(p)$. Then $X \times V$ is f.open set and $g(p) q X \times V$. Since g is f.θ-continuous, there exists f.open q-nbd U of p such that $g(\overline{U}) \leq \overline{X \times V} = X \times \overline{V}$. Also since g is graph of f, we have $f(\overline{U}) \leq \overline{V}$. Hence f is f.θ-continuous.

Theorem 3.5. Let X, Y and Z be fts's and $f: X \longrightarrow Y$ and $g: Y \longrightarrow Z$ be f.θ-continuous. Then the composite mapping $g \circ f: X \longrightarrow Z$ is f.θ-continuous.

Proof. Let p be any f.point in X and N be any f.θ-nbd of $g(f(p))$. Since g is f.θ-continuous, $g^{-1}(N)$ is f.θ-nbd of $f(p)$. Also since f is f.θ-continuous, $f^{-1}(g^{-1}(N))$ is f.θ-nbd of p. But $(g \circ f)^{-1}(N) = f^{-1}(g^{-1}(N))$. Therefore $g \circ f$ is f.θ-continuous.

Definition 3.5 [10]. f is said to be f.almost quasi-compact if it is onto and V is f.open in Y whenever $f^{-1}(V)$ is f.regularly open in X.

Theorem 3.6. Let $f: X \longrightarrow Y$ be onto. The following conditions are equivalent:

(a) f is f.almost quasi-compact.

(b) V is f.closed in Y for every f.regularly closed $f^{-1}(V)$.

(c) $f(U)$ is f.closed in Y for every f.regularly closed inverse set U.

(d) $f(U)$ is f.open in Y for every f.regularly open inverse set U.

Proof. (a)⇒(b): Let $f^{-1}(V)$ be f.regularly closed in Y. Then $f^{-1}(V')=$■ $f^{-1}(V')$ is f.regularly open in X. By (a), V' is f.open in Y. Hence V is f.closed in Y.

(b)⇒(c): Let U be f.regularly closed inverse set. Since $f^{-1}(f(U))=U$ is f.regularly closed in X, $f(U)$ is f.closed in Y.

(c)⇒(d): Let U be f.regularly open inverse set. Then U' is f.regularly closed. Since f is onto and U is inverse set, $f(U)'=f(f^{-1}(f(U)'))=f(U')$. Thus $f^{-1}(f(U'))=f^{-1}(f(U)')=U'$. That is, U' is inverse set. By (c), $f(U')=f(U')$ is f.closed. Hence $f(U)$ is f.open set.

(d)⇒(a): Let $f^{-1}(U)$ is f.regularly open. Since f is onto, $f^{-1}(U)$ is f.regularly open invers set. By (d), $f(f^{-1}(U))=U$ is f.open. Thus f is almost quasi-compact.

Definition 3.6 [10]. f is said to be

(a) f.almost open if the image of every f.regulary open set of X is f.open in Y.

(b) f.almost closed if the image of every f.regulary closed set of X is f.closed in Y.

If f:X⟶Y is bijective, by Nanda [10], then the following statements are equivalent:

(a) f is f.almost open.

(b) f is f.almost closed.

(c) f is f.almost quasi-compact.

(d) f^{-1} is f.almost continuous.

Definition 3.7 [9]. A fts X is said to be

(a) f.almost regular if for each f.regularly open set V and each f.point pqV, there exists a f.regularly open set U such that pq$U \langle \bar{U} \leq V$.

(b) f.semi-regular if for each f.open set V and each f.point pqV, there exists a f.open set U such that pq$V \leq (\bar{V})^{\circ} \leq V$.

Theorem 3.7. A fts X is f.almost regular if and only if $[A]^{-}=[A]_{\theta}$ for every f.set A of X.

Proof. In [9], $[A]_\theta \geq [A]$. Thus we show that $[A]_\theta \leq [A]_\delta$. Let $p \notin [A]_\delta$. Then there exists a f.regularly open q-nbd V of p such that $V \not q A$. Since X is f.almost regular, there exists a f.regularly open set U such that $pqU \leq \bar{U} \leq V$. Then $V \not q A \Rightarrow A \leq V' \leq (\bar{U})' \Rightarrow \bar{U} \not q A \Rightarrow p \notin [A]_\theta$.

Conversely, let V be a f.regularly open set and p be a f.point with pqV. Then since V' is f.δ-closed in X, $p \notin V' = [V']_\delta = [V']_\theta$. Thus there exists f.open q-nbd U of p such that $\bar{U} \not q V'$, so that $(\bar{U})^\circ \leq V^\circ \leq V$. Putting $G = (\bar{U})^\circ$, then G is f.regularly open set in X such that $pqG \leq \bar{G} \leq \bar{U} \leq V$. Hence X is f.almost regular.

Theorem 3.8. A fts X is f.almost regular if and only if $\bar{A} = [A]_\theta$ for every f.semi-open set A of X.

Proof. follows easily by virtue of Lemma 3.1 and Theorem 3.7.

Theorem 3.9. A fts X is f.semi-regular if and only if $\bar{A} = [A]_\delta$ for each f.set A of X.

Proof. The necessary condition is sufficient to show that $[A]_\delta \leq \bar{A}$. Let $p \notin \bar{A}$. Then there exists a q-nbd V of p such that $V \not q A$. Since X is f.semi-regular, there is a f.open set U such that $pqU \leq (\bar{U})^\circ \leq V$. Then $V \not q A \Rightarrow (\bar{U})^\circ \leq V \leq A' \Rightarrow (\bar{U})^\circ \not q A$ $\Rightarrow p \notin [A]_\delta$.

Conversely, let V be f.open set and p be any f.point with pqV. Since V' is f.closed set, $p \notin \overline{V'} = [V']_\delta$. Then there exists a f.open q-nbd U of p such that $(\bar{U})^\circ \not q V'$, so that $(\bar{U})^\circ \leq V$. Hence X is f.semi-regular.

Theorem 3.10. If $f: X \longrightarrow Y$ is a f.weakly continuous and Y is f.almost regular, then f is f.almost continuous.

Proof. Let p be a f.point in X and let M be a f.open q-nbd of $f(p)$. Since Y is f.almost regular, there exists a f.regularly open set N such that $f(p)qN \leq \bar{N} \leq (\bar{M})^\circ$. By f.weakly continuity of f, there exists a f.open q-nbd U of p such that $p q U$ and $f(U) \leq \bar{N} \leq (\bar{M})^{o}$. Thus f is f.almost continuous.

By the above theorem, the following corollary is easily obtained.

Corollary 3.11. Let $f: X \longrightarrow Y$ be a mapping. If Y is f.almost regular, then the following are equivalent:

(a) f is f.weakly continuous.

(b) f is f.θ-continuous.

(c) f is f.almost continuous.

Theorem 3.12. If $f: X \longrightarrow Y$ is a f.weakly continuous, f.almost open, and X is f.semi-regular, then f is f.almost continuous.

Proof. Let p be a f.point in X and V be any f.open q-nbd of $f(p)$. Since f is f.weakly continuous, there exists a f.open q-nbd U of p such that $f(U) \leq \bar{V}$. By f.semi-regularity of X, there exists a f.open set W such that $p q W$ and $W \leq (\bar{W})^{o} \leq U$. Since f is f.almost open and $(\bar{W})^{o}$ is f.regularly open, $f(W) \leq f((W)^{o}) \leq (\bar{V})^{o}$. Thus f is f.almost continuous.

Theorem 3.13. If $f: X \longrightarrow Y$ is a f.almost open and f.θ-continuous, then f is f.almost continuous.

Proof. Let p be a f.point in X and V be any f.open q-nbd of $f(p)$. Then since f is f.θ-continuous, there exists a f.open q-nbd U of p such that $f(\bar{U}) \leq \bar{V}$. Since f is f.almost open and $(\bar{U})^{o}$ is f.regularly open, $f((\bar{U})^{o})$ is f.open. Thus we have $f(U) \leq f((\bar{U})^{o}) \leq (f(\bar{U}))^{o}$. Therefore $f(U) \leq (\bar{V})^{o}$. This shows that f is f.almost continuous.

By the theorem 3.12 and Theorem 3.13, the following corollary is easily obtained.

Corollary 3.14. Let $f: X \longrightarrow Y$ be f.almost open mapping.

(a) f is f.almost continuous if and only if f is f.θ-continuous.

(b) Let X be f.semi-regular. Then f is f.almost continuous if and only if f is f.weakly continuous.

Thus, if f is f.almost open and X is f.semi-regular, then f.almost continuity, f.θ-continuity and f.weakly continuity are equivalent.

Theorem 3.15. Let $f:X\longrightarrow Y$ be f.θ-continuous, f.almost open, f.almost closed and onto. If X is f.almost regular, then Y is also f.almost regular.

Proof. Let $f(p)$ be a f.point in Y and B be any f.regularly open set such that $f(p)qB$. By theorem 3.13 and theorem 3.5 in [8], $f^{-1}(B)$ is a f.regularly open set of X such that $pqf^{-1}(B)$. Since X is f.almost regular, there exists a f.regularly open set U such that $pqU\leq\overline{U}\leq f^{-1}(B)$. Since f is f.almost closed, $f(\overline{U})$ is f.closed. Hence $\overline{f(U)}\leq f(\overline{U})$ and $f(p)qf(U)$. Putting $V=(\overline{f(U)})^{\circ}$, then V is f.regularly open set such that $f(p)qV$ and $\overline{V}=\overline{((\overline{f(U)})^{\circ})}\leq\overline{f(U)}\leq f(\overline{U})\leq B$. Thus Y is f.almost regular.

Definitions 3.8 [13]. A fts X is said to be f.Urysohn if for any distinct f.points p and r in X(i,e., satisfying $x_p\neq x_r$), there exist f.open sets U and V scuh that pqU, rqV and $\overline{U}\cap\overline{V}=\phi$.

Theorem 3.16. Let $f:X\longrightarrow Y$ be f.θ-continuous and one-to-one. If Y is f.Urysohn space, then X is also f.Urysohn.

Proof. Let p and r be distinct f.points in X. Then $f(p)$ and $f(r)$ are distinct f.points in Y. By f.Urysohn property of Y, there exist f.open sets V_1 and V_2 in Y such that $f(p)qV_1$, $f(r)qV_2$ and $\overline{V_1}\cap\overline{V_2}=\phi$. Since f is f.θ-continuous, there exist f.open sets U_1 and U_2 such that pqU_1, rqU_2, $f(\overline{u_1})\leq\overline{V_1}$ and $f(\overline{U_2})\leq\overline{V_2}$. Since f is one-to-one,

$$\overline{U_1}\cap\overline{U_2}=f^{-1}(f(\overline{U_1}))\cap f^{-1}(f(\overline{U_2}))$$
$$=f^{-1}(f(\overline{U_1})\cap f(\overline{U_2}))\leq f^{-1}(V_1\cap V_2)=\phi.$$

Thus X is f.Urysohn.

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