Journal of Liberal Arts and Sciences.
Korea Maritime University. Vol. 6. 1998

Extrapolation Values of Quasiparticle Energies in Fractional Quantum Hall Systems

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Abstract

A fractionally filled electron Landau level can be transformed into an integral number of filled Composite Fermion(CF) Landau levels. A Fermi liquid picture is applied to Laughlin incompressible liquid state. Excited states are described by n_{QE} and n_{QH} , the number of quasielectron and quasihole CF excitations. The energies of large numbers of quasielectrons or quasiholes are investigated by extrapolating from a small number of electrons on a spherical surface. This results was also compared with the self energy correction results suggested by Haldane and Rezayi.

The fractional quantum Hall effect(FQHE) is based largely on Laughlin's theory for the primary filling fractions $\nu = 1/m$, where m in an odd integer. Laughlin's picture was obtained by exact numerical diagonalization of the full interacting Hamiltonian for a small number of electrons (N=5,6,7,...) constrained to move on a spherical surface in a radial magnetic field[1]. The total flux $4\pi R^2 B$ through the sphere of radius R is equal to an integer, 2S, times the quantum of flux, $\Phi_0 = hc/e$. The eigenvalues of kinetic energy are given by $E_l = (\frac{hw_c}{4\pi S})\,[l(l+1) - S^2]$, where the angular momentum eigenvalue l can take on the values S , $S+1$, $S+2$,... Clearly the lowest Landau level has $l = S$, and the nth excited Landau level has $l = S+n$. There are $2S+1$ degenerate single particle states in the lowest Landau level. For the Laughlin $\nu = 1/m$ state 2S has the value $m(N-1)$, so that for N=8, the $\nu = 1/3$ state has 2S=21. Numerical calculations for up to ten electrons have been carried out for special values of the filling factor ν [2,3].

The energy spectrum obtained in this numerical calculations gives a set of energy values for each value of the total angular momentum L. An illustrative case is the spectrum for an eight electron system for 2S=21, 20, 19, 18, 17, and 16. The value 2S=21 corresponds to the Laughlin

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liquid state. The ground state is an L=0 single, and it is well separated from a low lying set of excited states. For values of 2S close to the value for a Laughlin state, we can write $2S=2S_{\nu=\frac{1}{3}}+n_{QH}-n_{QE}$, where n_{QE} and n_{QH} are the number of quasielectron and number of quasihole excitations, respectively. Thus the systems with 2S=20 through 16 correspond to values of $n_{QH}-n_{QE}$ going from -1 to -5. The lowest energy sectors of these spectra have $n_{QH}=0$. Thus $n_{QE}=1$ through 5 for these five values of 2S. The first excited sector is expected to contain one additional QE - QH pair. It would be useful to be able to determine the energies and angular momenta of the states in the low lying energy sectors without having to perform very large numerical diagonalizations. It may be possible to accomplish this by using the numerical calculations to construct a Fermi liquid model of the CF excitations.[4,5,6]

In the Composite Fermion medel introduced by Jain[7], a fictitious flux $2p\Phi_0$ (where p is an integer) is attached to each electron. For a filling factor ν the applied magnetic field $\vec{B}$ satisfies $B \cdot A/N=\Phi_0 / \nu$, where A is the area of the sample. For the $\nu=1/3$ state we choose p = -1, so that the fictitious magnetic field $\vec{b}$ produced by the fictitious flux is oriented opposite to the applied field $\vec{B}$. The fictitious charge is taken equal to the charge on the electron, then each CF experiences a mean field $\vec{B}^*=\vec{B}+\vec{b}=\vec{B}/3$. In the spherical geometry this transforms the value of 2S into an effective flux $2S^*=2S^*_{\nu=1}+n_{QH}-n_{QE}$ acting on the Composite Fermions[8].

For an N particle system $2S^*_{\nu=1}=N-1$. S^* plays the same role for the CF that S plays for the original electron. The angular momentum of a particle in the lowest CF Landau level must be equal to S^*, giving for the angular momentum of a quasihole excitation

$$l_{QH}=S^*=\frac{1}{2}(N+n_{QH}-n_{QE}-1). \tag{1}$$

Because the quasielectron excitations are Composite Fermions in the normally empty first excited CF Landau level $l_{QE}=S^*+1=l_{QH}+1$.

The energy of a QE - QH pair is just the CF cyclotron energy. However, a single QE or QH must interact with the Laughlin "vacuum state". In this case it can be taken the results of exact numerical diagonalization for N electrons on a sphere as "experimental" data.

For example, the Laughlin condensed state of an eight electron system occurs at 2S=21 and has angular momentum L=0. A single quasielectron is present at 2S=20 with L=4, a single quasihole occurs at 2S=22 with L=4. The energies ε_{QE} and ε_{QH} are simply the difference between the minimum energy, E(L=0), of the Laughlin condensed state at 2S=21 and the minimum energy E(L=4) at 2S=20(QE) or at 2S=22(QH). In considering the quasiparticle(QP is either QE or QH) states, we keep the number of electrons and the radius of the sphere constant, but change the value of 2S. We can obtain the QE and QH energy as a

function of N^{-1}. Extrapolation to $N^{-1} \rightarrow 0$ should eliminate finite size effects and give values of ε_{QE} and ε_{QH} appropriate for large systems[9].

For states with more than a single quasiparticle excitation, we can determine the allowed values of the total angular momentum by simple addition of the angular monenta of the individual quasielectron and quasiholes treated as distinquishable sets of Fermions.

In the Landau theory the energy of an interacting electron system is given by the sum of the energies of the individual quasiparticles plus a term representing the interaction of the quasiparticles with one another. Thus far we have neglected the interaction term. Thus the energy of a state containing n_{QE} quasielectrons and n_{QH} quasiholes would be simply $n_{QE}\ \varepsilon_{QE} + n_{QH}\ \varepsilon_{QH}$. The energy of a state containing n_{QE} quasielectrons and n_{QH} quasiholes can be written as

$$E = E_0 + \sum_{QP} \varepsilon_{QP} n_{QP} + \frac{1}{2} \sum_{QP,QP'} V_{QP,QP'}\ (L_{QP,QP'}) n_{QP} n_{QP'} \tag{2}$$

The last term represents the sum of the energy changes caused by the interaction of pairs of quasiparticles whose total angular momentum is $L_{QP,QP'}$. The QP-QP interaction function $V_{QP\ QP'}(\mathrm{L})$ can be obtained by comparing the exact numerical energies for the states containing a single pair of quasiparicles (2QE, 2QH or 1QE + 1QH) with the sum of the individual quasiparticle energies. In the figure we plot the ε_{QE} and ε_{QH} which obtained from the numerical calculations as a function of the inverse of the number of electrons. We believe that systems with large numbers of electrons can be treated using the phenomeno-logical V_{QP-QP} to determine how the states in the low energy sectors are affected by QP – QP interactions.

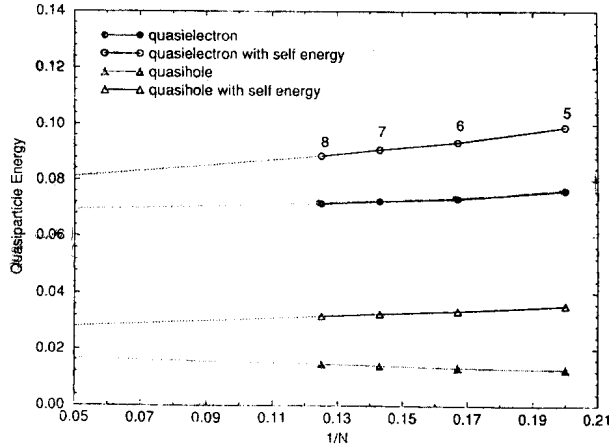

FIG. **ε_{QE} and ε_{QH} obtained from the numerical calculations as a function of the inverse of the number of electrons. The open symbols includes the "self-energy" correction suggested by Haldane and Rezayi. Energies are in units of e^2 / l_0, where l_0 is the magnetic length for the field appropriate to the Laughlin $\nu = 1/3$ state.**

The problem of determining the relative or total angular momentum of pairs of quasipriticles when you know that the total angular momentum of three, four or more quasiparticles is equal to some value L is non-trivial. For the Fermi liquid theory approach it is conceptually simpler to use semiclassical quantization and to describe quasiparticle coordinates in terms of energy, E, and a "time along orbit", S, where the motion is a periodic function of S with period $2\pi/w_c$. The interaction of a pair of quasiparticles (ε', S') and (ε, S) will then be a periodic function of $S'-S$ and will depend upon whether ε and ε' describe QE or QH excitations. The Landau interaction function $f_{QP,QP'}(S'-S)$ can be thought of as a two by two matrix with diagonal elements QE - QE and QH - QH, and off-diagonal elements QE - QH. The function can be expanded in Fourier series, since it is periodic in $S'-S$, and the Fourier coefficients will play a significant role in describing excited states.

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