to assure good operating condition, since the faulty operation of an engine is deeply related to public health.

2. Any symptoms of deviation from normal condition will be a clue to find out the defective oeprating units. But guess work must never be done. Check with testers correctly.
3. All engines must be provided with following papers positively to insure 100% efficient tune-up jobs.
   a. The trouble shooting chart
   b. The job analysis chart
   c. The troubles and tune-up recording cards
   d. The scientific test report
4. For the educational purpose, the progressive chart also should be added.
5. To confirm the effiecicny of the trouble shooting chart, 100 trainees were tested and its consequence showed that the trainees trained with the chart were superior to those trained without the chart, in understanding by 34%, in accuracy by 46% and in time to locate the troubles by 51%. Therefore, the trouble shooting chart is not only recommended for the effiicient tune-up job, but also for the educational purposes.

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Automotive Repair Manuals

Testing Instrument Maker's Instruction Books

$\mu(\theta)=\log_e\left|\cot\frac{\theta}{2}\right|$와 $\lambda(\theta)=\log_e\left|\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)\right|$의 双曲線函數와 그 應用에 關하여

蔡 亨 鎔

On the relationship between the Hyperbolic functions of $\mu(\theta)=\log_e\left|\cot\frac{\theta}{2}\right|$, $\lambda(\theta)=\log_e\left|\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)\right|$ and the application.

Chai Hyung-yong

Abstract

It is stated the changing method of trigonometrical functions to Hyperbolic functions, and the result was expressed on the generaligation table.

Using this table to put in practice in spherical trigono.-solution and used this result to navigational triangle solution too。

It is easily solvable the spherical trigono. and navigational triangle-solution if drawing up a table of $\mu(\theta)=\log_e\left|\cot\frac{\theta}{2}\right|$ for the value of θ, and a table of Hyprerbolic fuctions for the value of this result.

<目 次>

1. 序 言

1-1. $\sinh x=\cot\theta\left(-\frac{\pi}{2}<\theta<\frac{\pi}{2}\right)$라 하면, $\cosh x=\operatorname{cosec}\theta$, $\tanh x=\cos\theta$ 임을 밝혀라. 라는 問題에서 $x=\log_e\cot\frac{\theta}{2}$라 놓으면 双曲線函數를 三角函數로 變換할 수 있다.

1-2. $\cos a=\cos a\cdot\cos c+\sin a\cdot\sin c\cdot\cos A$ 에서

$$\cos A=\frac{\cot\frac{a}{2}-\tan\frac{a}{2}}{2\cot\frac{c}{2}}$$ 임을 證明하라. 라는 問題에서

주어진 式은 斜角球面三角形의 第一 cosine 法則에서 右邊의 b를 a로 놓은 것이며,

$$\cos A = \frac{\cot\frac{a}{2} - \tan\frac{a}{2}}{2\cot\frac{c}{2}} = \frac{\sinh\mu(a)}{e^{\mu(c)}}$$ 로 쓸수 있음을 알 수 있다.

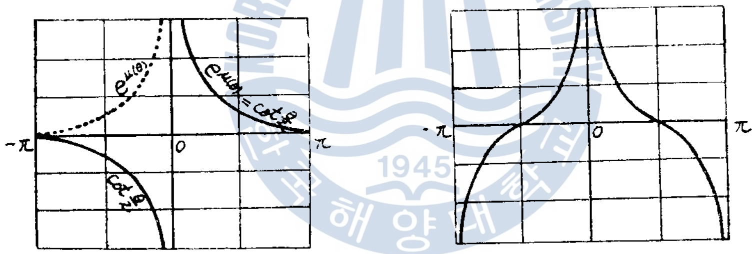
$$\left\{ \mu(\theta) = \log\cot\frac{\theta}{2} \text{라 할때} \right\}$$

이것을 着眼하면 球面三角形에서 三邊을 알고 三角을 求하는 問題를 $e^{\mu(\theta)}$ **或은** $\sinh\mu(\theta)$**인 函數**로 解할 수 있을 것이 아닌가 하고, 本章을 엮어본 것이다. 따라서 이것을 天文三角形 解法에 應用 해 보려는 것을 目的으로 하였다.

2. 本 論

2-1. $\cot\frac{\theta}{2}$와 $\mu(\theta)$

$$\mu(\theta) = \log_e\left|\cot\frac{\theta}{2}\right| \quad \text{즉} \quad e^{\mu(\theta)} = \left|\cot\frac{\theta}{2}\right|$$



$\cot\frac{\theta}{2}$ $\qquad\qquad$ $\mu(\theta) = \log\left|\cot\frac{\theta}{2}\right|$

$$\cot\frac{\theta}{2} = \frac{\cos\frac{1}{2}\left(\frac{\pi}{2}-\theta\right) + \sin\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)}{\cos\frac{1}{2}\left(\frac{\pi}{2}-\theta\right) - \sin\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)}$$

$$= \frac{\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)+1}{\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)-1} = \frac{1+\cos\theta}{\sin\theta} = \operatorname{cosec}\theta + \cot\theta$$

$$= \frac{\sin\theta}{1-\cos\theta} = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$$

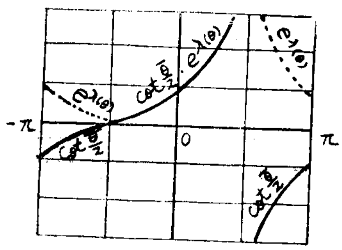
$$= \pm\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} \qquad \theta : \begin{cases} \text{I, II, 상한}(+) \\ \text{III, IV, 상한}(-) \end{cases}$$

$$e^{\mu(\theta)} = \pm\cot\frac{\theta}{2} \qquad \theta : \begin{cases} \text{I, II, 상한}(+) \\ \text{III, IV, 상한}(-) \end{cases}$$

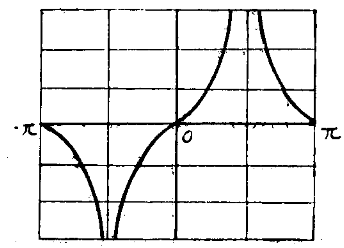
$$\begin{cases} \mu(-\theta) = \mu(\theta) \\ \mu(\pi\pm\theta) = -\mu(\theta) \end{cases}$$

2-2. $\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)$ 와 $\lambda(\theta)$

$$\lambda(\theta)=\log_e\left|\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)\right| \qquad \text{즉} \quad e^{\lambda(\theta)}=\left|\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)\right|$$



$\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)$



$\lambda(\theta)=\log_e\left|\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)\right|$

$$\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)=\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}=\tan\frac{1}{2}\left(\frac{\pi}{2}+\theta\right)$$

$$=\frac{\cot\frac{\theta}{2}+1}{\cot\frac{\theta}{2}-1}=\frac{1+\sin\theta}{\cos\theta}=\sec\theta+\tan\theta$$

$$=\frac{\cos\theta}{1-\sin\theta}=\frac{1}{\sec\theta-\tan\theta}$$

$$=\pm\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \qquad \theta:\begin{cases}\text{I, IV, 상한}(+)\\ \text{II, III, 상한}(-)\end{cases}$$

$$e^{\lambda(\theta)}=\pm\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right) \qquad \theta:\begin{cases}\text{I, IV, 상한}(+)\\ \text{II, III, 상한}(-)\end{cases}$$

$$\begin{cases}\lambda(-\theta)=-\lambda(\theta)\\ \lambda(\pi-\theta)=\lambda(\theta)\\ \lambda(\pi+\theta)=-\lambda(\theta)\end{cases}$$

2-3. $\mu(\theta)$와 $\lambda(\theta)$

$$\mu\left(\frac{\pi}{2}-\theta\right)=\lambda(\theta) \qquad \mu\left(\frac{\pi}{2}+\theta\right)=-\lambda(\theta)$$

$$\lambda\left(\frac{\pi}{2}-\theta\right)=\mu(\theta) \qquad \lambda\left(\frac{\pi}{2}+\theta\right)=\mu(\theta)$$

$$\frac{e^{\mu(\theta)}+1}{e^{\mu(\theta)}-1}=\begin{cases}e^{\lambda(\theta)} & \text{I. 상한}\\ -e^{\lambda(\theta)} & \text{II. 〃}\\ -e^{-\lambda(\theta)} & \text{III. 〃}\\ e^{-\lambda(\theta)} & \text{IV. 〃}\end{cases} \qquad \frac{e^{\lambda(\theta)}+1}{e^{\lambda(\theta)}-1}=\begin{cases}e^{\mu(\theta)} & \text{I. 상한}\\ e^{-\mu(\theta)} & \text{II. 〃}\\ -e^{-\mu(\theta)} & \text{III. 〃}\\ -e^{\mu(\theta)} & \text{IV. 〃}\end{cases}$$

$$\log_e\coth\frac{\mu(\theta)}{2}=\log\left|\frac{e^{\mu(\theta)}+1}{e^{\mu(\theta)}-1}\right| \qquad \log_e\coth\frac{\lambda(\theta)}{2}=\log\left|\frac{e^{\lambda(\theta)}+1}{e^{\lambda(\theta)}-1}\right|$$

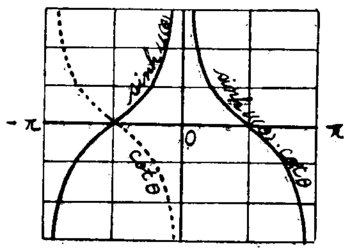
$$\log_e\coth\frac{\mu(\theta)}{2}=\begin{cases}\lambda(\theta) & \text{I, II. 상한}\\ -\lambda(\theta) & \text{III, IV. 〃}\end{cases} \qquad \log_e\coth\frac{\lambda(\theta)}{2}=\begin{cases}\mu(\theta) & \text{I, IV. 상한}\\ -\mu(\theta) & \text{II, III. 〃}\end{cases}$$

$\mu(\theta)$와 $\lambda(\theta)$는 θ가 I, III. 상한이면 同符號, II, IV. 〃 異符號

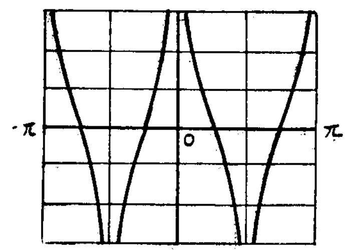
2-4. $\mu(\theta)$ 및 $\lambda(\theta)$의 **双曲線函數**

$$\sinh\ \mu(\theta)=\frac{e^{\mu(\theta)}-e^{-\mu(\theta)}}{2}=\frac{e^{2\mu(\theta)}-1}{2e^{\mu(\theta)}}$$

$$=\frac{\cot^2\frac{\theta}{2}-1}{\pm 2\cot\frac{\theta}{2}}=\pm\cot\theta \qquad \theta:\begin{cases}\text{I, II. 상한}(+)\\ \text{III, IV. 상한}(-)\end{cases}$$



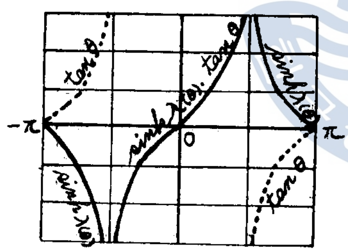
sinh $\mu(\theta)$



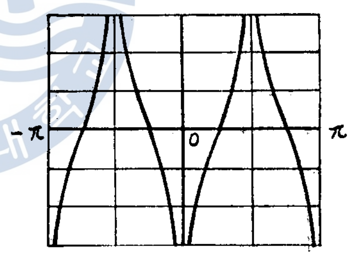
log | sinh $\mu(\theta)$ |

$$\sinh\lambda(\theta)=\frac{e^{\lambda(\theta)}-e^{-\lambda(\theta)}}{2}=\frac{e^{2\lambda(\theta)}-1}{2e^{\lambda(\theta)}}$$

$$=\frac{\cot^2\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)-1}{\pm 2\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)}=\pm\tan\theta \qquad \theta:\begin{cases}\text{I, IV. 상한}(+)\\ \text{II, III. 상한}(-)\end{cases}$$



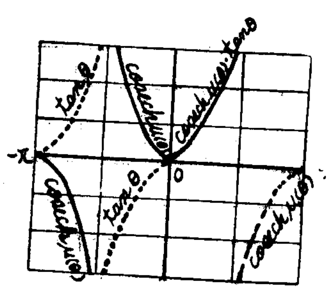
sinh $\lambda(\theta)$



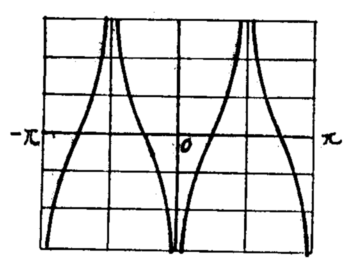
log | sinh $\lambda(\theta)$ |

$$\text{cosech}\ \mu(\theta)=\frac{2}{e^{\mu(\theta)}-e^{-\mu(\theta)}}=\frac{2e^{\mu(\theta)}}{e^{2\mu(\theta)}-1}$$

$$=\frac{\pm 2\cot\frac{\theta}{2}}{\cot^2\frac{\theta}{2}-1}=\pm\tan\theta \qquad \theta:\begin{cases}\text{I, II. 상한}(+)\\ \text{III, IV. 상한}(-)\end{cases}$$



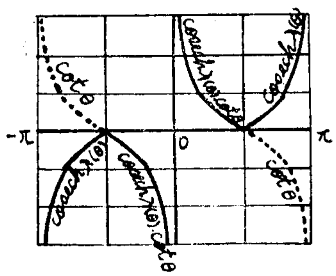
cosech $\mu(\theta)$



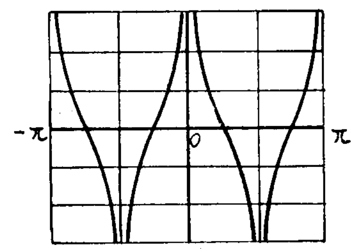
log cosech $\mu(\theta)$

$$\operatorname{cosech} \lambda(\theta) = \frac{2}{e^{\lambda(\theta)} - e^{-\lambda(\theta)}} = \frac{2e^{\lambda(\theta)}}{e^{2\lambda(\theta)} - 1}$$

$$= \frac{\pm 2 \cot \frac{1}{2}\left(\frac{\pi}{2} - \theta\right)}{\cot^2 \frac{1}{2}\left(\frac{\pi}{2} - \theta\right) - 1} = \pm \cot \theta \qquad \theta : \begin{cases} \text{I, IV. 상한}(+) \\ \text{II, III. 상한}(-) \end{cases}$$



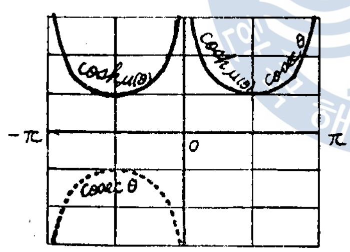
cosech $\lambda(\theta)$



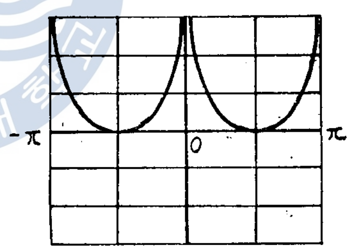
log | cosech $\lambda(\theta)$ |

$$\cosh \mu(\theta) = \frac{e^{\mu(\theta)} + e^{-\mu(\theta)}}{2} = \frac{e^{2\mu(\theta)} + 1}{2e^{\mu(\theta)}}$$

$$= \frac{\cot^2 \frac{\theta}{2} + 1}{\pm 2 \cot \frac{\theta}{2}} = \pm \operatorname{cosec} \theta \qquad \theta : \begin{cases} \text{I, II 상한}(+) \\ \text{III, IV 상한}(-) \end{cases}$$



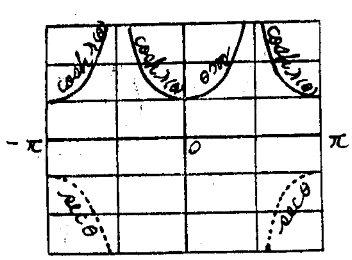
cosh $\mu(\theta)$



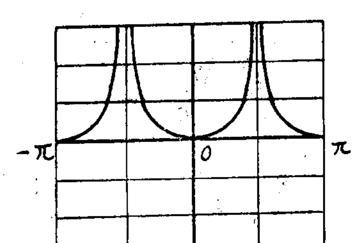
log | cosh $\mu(\theta)$ |

$$\cosh \lambda(\theta) = \frac{e^{\lambda(\theta)} + e^{-\lambda(\theta)}}{2} = \frac{e^{2\lambda(\theta)} + 1}{2e^{\lambda(\theta)}}$$

$$= \frac{\cot^2 \frac{1}{2}\left(\frac{\pi}{2} - \theta\right) + 1}{\pm 2 \cot \frac{1}{2}\left(\frac{\pi}{2} - \theta\right)} = \pm \sec \theta \qquad \theta : \begin{cases} \text{I, IV. 상한}(+) \\ \text{II, III. 상한}(-) \end{cases}$$



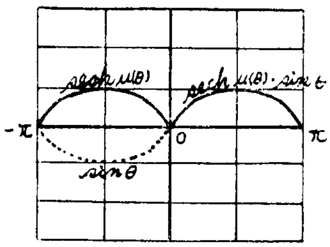
cosh $\lambda(\theta)$



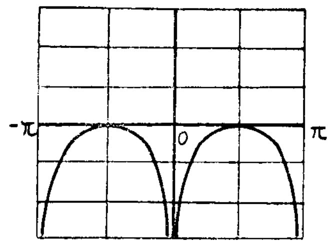
log | cosh $\lambda(\theta)$ |

$$\operatorname{sech}\mu(\theta)=\frac{2}{e^{\mu(\theta)}+e^{-\mu(\theta)}}=\frac{2e^{\mu(\theta)}}{e^{2\mu(\theta)}+1}$$

$$=\frac{\pm 2\cot\dfrac{\theta}{2}}{\cot^2\dfrac{\theta}{2}+1}=\pm\sin\theta \qquad \theta:\begin{cases}\text{I, II. 상한}(+)\\ \text{III, IV. 상한}(-)\end{cases}$$



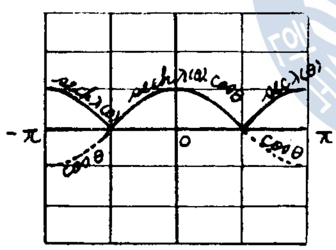
sech $\mu(\theta)$



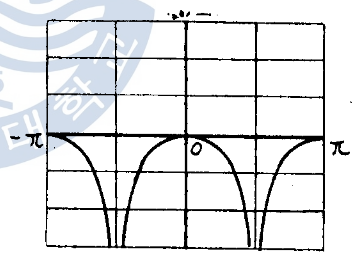
log | sech $\mu(\theta)$ |

$$\operatorname{sech}\lambda(\theta)=\frac{2}{e^{\lambda(\theta)}+e^{-\lambda(\theta)}}=\frac{2e^{\lambda(\theta)}}{e^{2\lambda(\theta)}+1}$$

$$=\frac{\pm 2\cot\dfrac{1}{2}\left(\dfrac{\pi}{2}-\theta\right)}{\cot^2\dfrac{1}{2}\left(\dfrac{\pi}{2}-\theta\right)+1}=\pm\cos\theta \qquad \theta:\begin{cases}\text{I, IV. 상한}(+)\\ \text{II, III. 상한}(-)\end{cases}$$



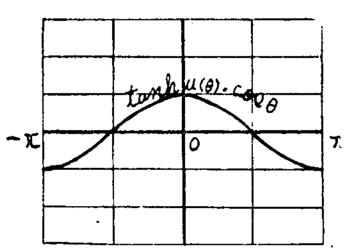
sech $\lambda(\theta)$



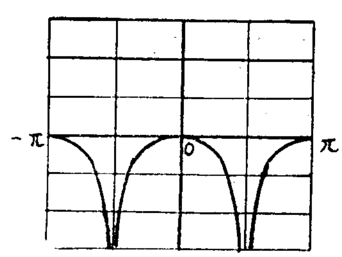
log | sech $\lambda(\theta)$ |

$$\tanh\mu(\theta)=\frac{e^{\mu(\theta)}-e^{-\mu(\theta)}}{e^{\mu(\theta)}+e^{-\mu(\theta)}}=\frac{e^{2\mu(\theta)}-1}{e^{2\mu(\theta)}+1}$$

$$=\frac{\cot^2\dfrac{\theta}{2}-1}{\cot^2\dfrac{\theta}{2}+1}=\cos\theta$$



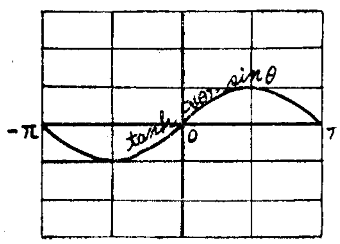
tanh $\mu(\theta)$



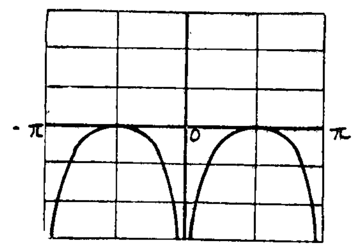
log | tanh $\mu(\theta)$ |

$$\tanh \lambda(\theta)=\frac{e^{\lambda(\theta)}-e^{-\lambda(\theta)}}{e^{\lambda(\theta)}+e^{-\lambda(\theta)}}=\frac{e^{2\lambda(\theta)}-1}{e^{2\lambda(\theta)}+1}$$

$$=\frac{\cot^2\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)-1}{\cot^2\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)+1}=\sin\theta$$



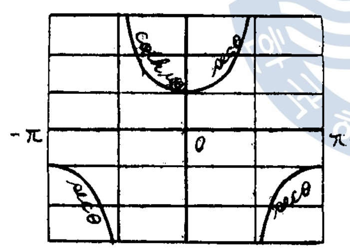
tanh $\lambda(\theta)$



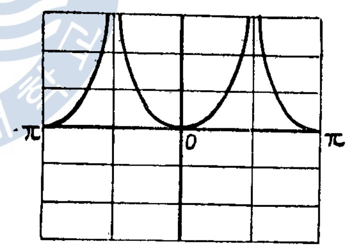
log | tanh $\lambda(\theta)$ |

$$\coth \mu(\theta)=\frac{e^{\mu(\theta)}+e^{-\mu(\theta)}}{e^{\mu(\theta)}-e^{-\mu(\theta)}}=\frac{e^{2\mu(\theta)}+1}{e^{2\mu(\theta)}-1}$$

$$=\frac{\cot^2\frac{\theta}{2}+1}{\cot^2\frac{\theta}{2}-1}=\sec\theta$$



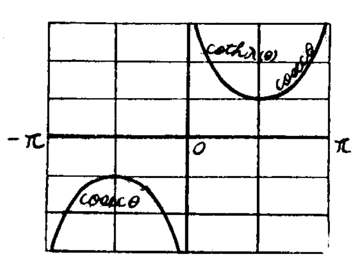
coth $\mu(\theta)$



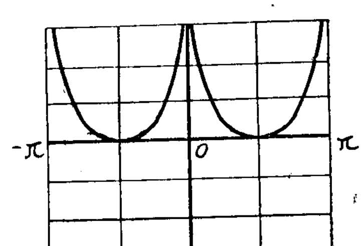
log | coth $\mu(\theta)$ |

$$\coth \lambda(\theta)=\frac{e^{\lambda(\theta)}+e^{-\lambda(\theta)}}{e^{\lambda(\theta)}-e^{-\lambda(\theta)}}=\frac{e^{2\lambda(\theta)}+1}{e^{2\lambda(\theta)}-1}$$

$$=\frac{\cot^2\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)+1}{\cot^2\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)-1}=\operatorname{cosec}\theta$$



coth $\lambda(\theta)$



log coth $\lambda(\theta)$

上述한 바와 같이 θ의 三角函數를 $\mu(\theta)$ 및 $\lambda(\theta)$의 双曲線函數로 變換할 수 있다.

총　괄

상　한	I	II	III	IV
$\sin\theta$	$\operatorname{sech}\mu(\theta)$ $\tanh\lambda(\theta)$	$\operatorname{sech}\mu(\theta)$ $\tanh\lambda(\theta)$	$-\operatorname{sech}\mu(\theta)$ $\tanh\lambda(\theta)$	$-\operatorname{sech}\mu(\theta)$ $\tanh\lambda(\theta)$
$\operatorname{cosec}\theta$	$\cosh\mu(\theta)$ $\coth\lambda(\theta)$	$\cosh\mu(\theta)$ $\coth\lambda(\theta)$	$-\cosh\mu(\theta)$ $\coth\lambda(\theta)$	$-\cosh\mu(\theta)$ $\coth\lambda(\theta)$
$\cos\theta$	$\tanh\mu(\theta)$ $\operatorname{sech}\lambda(\theta)$	$\tanh\mu(\theta)$ $-\operatorname{sech}\lambda(\theta)$	$\tanh\mu(\theta)$ $-\operatorname{sech}\lambda(\theta)$	$\tanh\mu(\theta)$ $\operatorname{sech}\lambda(\theta)$
$\sec\theta$	$\coth\mu(\theta)$ $\cosh\lambda(\theta)$	$\coth\mu(\theta)$ $-\cosh\lambda(\theta)$	$\coth\mu(\theta)$ $-\cosh\lambda(\theta)$	$\coth\mu(\theta)$ $\cosh\lambda(\theta)$
$\tan\theta$	$\operatorname{cosech}\mu(\theta)$ $\sinh\lambda(\theta)$	$\operatorname{cosech}\mu(\theta)$ $-\sinh\lambda(\theta)$	$-\operatorname{cosech}\mu(\theta)$ $-\sinh\lambda(\theta)$	$-\operatorname{cosech}\mu(\theta)$ $\sinh\lambda(\theta)$
$\cot\theta$	$\sinh\mu(\theta)$ $\operatorname{cosech}\lambda(\theta)$	$\sinh\mu(\theta)$ $-\operatorname{cosech}\lambda(\theta)$	$-\sinh\mu(\theta)$ $-\operatorname{cosech}\lambda(\theta)$	$-\sinh\mu(\theta)$ $\operatorname{cosech}\lambda(\theta)$

3. 球面三角形의 公式에 對한 應用

3－1. $\cos a=\cos b\cdot\cos c+\sin b\cdot\sin c\cdot\cos A$

$$\cos A=\frac{\cos a}{\sin b}\operatorname{cosec}c-\frac{\cos b}{\sin b}\cot c$$

$$=\frac{\cos\frac{1}{2}\left\{(a+b)+(a-b)\right\}}{\sin\frac{1}{2}\left\{(a+b)-(a-b)\right\}}\operatorname{cosec}c-\frac{\cos\frac{1}{2}\left\{(a+b)-(a-b)\right\}}{\sin\frac{1}{2}\left\{(a+b)-(a-b)\right\}}\cot c$$

$$=\frac{\cos\frac{1}{2}(a+b)\cdot\cos\frac{1}{2}(a-b)-\sin\frac{1}{2}(a+b)\cdot\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)\cdot\cos\frac{1}{2}(a-b)-\cos\frac{1}{2}(a+b)\cdot\sin\frac{1}{2}(a-b)}\operatorname{cosec}c$$

$$-\frac{\cos\frac{1}{2}(a+b)\cdot\cos\frac{1}{2}(a-b)+\sin\frac{1}{2}(a+b)\cdot\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)\cdot\cos\frac{1}{2}(a-b)-\cos\frac{1}{2}(a+b)\cdot\sin\frac{1}{2}(a-b)}\cot c$$

$$=\frac{\cot\frac{1}{2}(a+b)\cdot\cot\frac{1}{2}(a-b)-1}{\cot\frac{1}{2}(a-b)-\cot\frac{1}{2}(a+b)}\operatorname{cosec}c-\frac{\cot\frac{1}{2}(a+b)\cdot\cot\frac{1}{2}(a-b)+1}{\cot\frac{1}{2}(a-b)-\cot\frac{1}{2}(a+b)}\cot c$$

(i) $a>b,\ \ a+b<\pi$

$$\cos A=\frac{e^{\mu(a+b)+\mu(a-b)}-1}{e^{\mu(a-b)}-e^{\mu(a+b)}}\cosh\mu(c)-\frac{e^{\mu(a+b)+\mu(a-b)}+1}{e^{\mu(a-b)}-e^{\mu(a+b)}}\sinh\mu(c)$$

$$=\frac{e^{\frac{1}{2}\{\mu(a+b)+\mu(a-b)\}}-e^{-\frac{1}{2}\{\mu(a+b)+\mu(a-b)\}}}{e^{\frac{1}{2}\{\mu(a-b)-\mu(a+b)\}}-e^{-\frac{1}{2}\{\mu(a-b)-\mu(a+b)\}}}\cosh\mu(c)$$

$$-\frac{e^{\frac{1}{2}\{\mu(a+b)+\mu(a-b)\}}+e^{-\frac{1}{2}\{\mu(a+b)+\mu(a-b)\}}}{e^{\frac{1}{2}\{\mu(a-b)-\mu(a+b)\}}-e^{-\frac{1}{2}\{\mu(a-b)-\mu(a+b)\}}}\sinh\mu(c)$$

$$=\frac{\sinh\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}}{\sinh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\cosh\mu(c)-\frac{\cosh\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}}{\sinh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\sinh\mu(c)$$

$$=\frac{\sinh\left[\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}-\mu(c)\right]}{\sinh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\cdots\cdots(1)$$

(ii) $a>b,\ a+b>\pi$

$$\cos A=-\frac{e^{\mu(a+b)+\mu(a-b)}+1}{e^{\mu(a-b)}+e^{\mu(a+b)}}\cosh\mu(c)+\frac{e^{\mu(a+b)+\mu(a-b)}-1}{e^{\mu(a-b)}+e^{\mu(a+b)}}\sinh\mu(c)$$

$$=-\frac{e^{\frac{1}{2}\{\mu(a+b)+\mu(a-b)\}}+e^{-\frac{1}{2}\{\mu(a+b)+\mu(a-b)\}}}{e^{\frac{1}{2}\{\mu(a-b)-\mu(a+b)\}}+e^{-\frac{1}{2}\{\mu(a-b)-\mu(a+b)\}}}\cosh u(c)$$

$$+\frac{e^{\frac{1}{2}\{\mu(a+b)+\mu(a-b)\}}-e^{-\frac{1}{2}\{\mu(a+b)+\mu(a-b)\}}}{e^{\frac{1}{2}\{\mu(a-b)-\mu(a+b)\}}+e^{-\frac{1}{2}\{\mu(a-b)-\mu(a+b)\}}}\sinh\mu(c)$$

$$=\frac{\cosh\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}}{\cosh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\cosh\mu(c)+\frac{\sinh\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}}{\cosh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\sinh\mu(c)$$

$$=-\frac{\cosh\left[\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}-\mu(c)\right]}{\cosh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\cdots\cdots(2)$$

(iii) $a+b<\pi,\ a<b$

$$\cos A=\frac{\cosh\left[\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)-\mu(c)\right]\right.}{\cosh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\cdots\cdots(3)$$

(iv) $a+b>\pi,\ a<b$

$$\cos A=-\frac{\sinh\left[\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}-\mu(c)\right]}{\sinh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\cdots\cdots(4)$$

3-2 $\cos b=\cos c\ \cos a+\sin c\cdot\sin a\cdot\cos B$

$$\cos B=\frac{\cos b}{\sin a}\operatorname{cosec}c-\frac{\cos a}{\sin a}\cot c$$

$$=\frac{\cos\frac{1}{2}\left\{(b+a)+(b-a)\right\}}{\sin\frac{1}{2}\left\{(b+a)-(b-a)\right\}}\operatorname{cosec}c-\frac{\cos\frac{1}{2}\left\{(b+a)-(b-a)\right\}}{\sin\frac{1}{2}\left\{(b+a)-(b-a)\right\}}\cot c$$

$$=\frac{\cot\frac{1}{2}(b+a)\cdot\cot\frac{1}{2}(b-a)-1}{\cot\frac{1}{2}(b-a)-\cot\frac{1}{2}(b+a)}\operatorname{cosec}c-\frac{\cot\frac{1}{2}(b+a)\cdot\cot\frac{1}{2}(b-a)+1}{\cot\frac{1}{2}(b-a)-\cot\frac{1}{2}(b+a)}\cot c$$

$$=\frac{\cot\frac{1}{2}(a+b)\cdot\cot\frac{1}{2}(a-b)+1}{\cot\frac{1}{2}(a-b)+\cot\frac{1}{2}(a+b)}\operatorname{cosec} c-\frac{\cot\frac{1}{2}(a+b)\cdot\cot\frac{1}{2}(a-b)-1}{\cot\frac{1}{2}(a-b)+\cot\frac{1}{2}(a+b)}\cot c$$

$$a+b<\pi,\quad a<b,\ \cos B=\frac{\sinh\left[\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}-\mu(c)\right]}{\sinh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\ \cdots\cdots(5)$$

$$a+b<\pi,\quad a>b,\ \cos B=\frac{\cosh\left[\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}-\mu(c)\right]}{\cosh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\ \cdots\cdots(6)$$

$$a+b>\pi,\quad a<b,\ \cos B=-\frac{\cosh\left[\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}-\mu(c)\right]}{\cosh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\ \cdots\cdots(7)$$

$$a+b>\pi,\quad a>b,\ \cos B=-\frac{\sinh\left[\frac{1}{2}\left\{\mu(a+b)+\mu(a-b)\right\}-\mu(c)\right]}{\sinh\frac{1}{2}\left\{\mu(a-b)-\mu(a+b)\right\}}\ \cdots\cdots(8)$$

이것을 綜合하면 $\cos A=\tanh\mu(A)$, $\cos B=\tanh\mu(B)$, 이므로

$a+b<\pi$	$a<b$	$a>b$
$\dfrac{\sinh\left\{\frac{\mu(a+b)+\mu(a-b)}{2}-\mu(c)\right\}}{\sinh\frac{\mu(a-b)-\mu(a+b)}{2}}$	$=\begin{cases}\cos B\\ \tanh\mu(B)\end{cases}$	$=\begin{cases}\cos A\\ \tanh\mu(A)\end{cases}$
$\dfrac{\cosh\left\{\frac{\mu(a+b)+\mu(a-b)}{2}-\mu(c)\right\}}{\cosh\frac{\mu(a-b)-\mu(a+b)}{2}}$	$=\begin{cases}\cos A\\ \tanh\mu(A)\end{cases}$	$=\begin{cases}\cos B\\ \tanh\mu(B)\end{cases}$
$a+b>\pi$		
$-\dfrac{\sinh\left\{\frac{\mu(a+b)+\mu(a-b)}{2}-\mu(c)\right\}}{\sinh\frac{\mu(a-b)-\mu(a+b)}{2}}$	$=\begin{cases}\cos A\\ \tanh\mu(A)\end{cases}$	$=\begin{cases}\cos B\\ \tanh\mu(B)\end{cases}$
$-\dfrac{\cosh\left\{\frac{\mu(a+b)+\mu(a-b)}{2}-\mu(c)\right\}}{\cosh\frac{\mu(a-b)-\mu(a+b)}{2}}$	$=\begin{cases}\cos B\\ \tanh\mu(B)\end{cases}$	$=\begin{cases}\cos A\\ \tanh\mu(A)\end{cases}$
$\dfrac{\sinh\mu(a)}{e^{\mu(c)}}$ (緖言 1−2와 同一함)	$a=b$	

吟味 $a=b$:
- A, a가 같은 상한이면 解 1.
- $A=a=\frac{\pi}{2}$ 이면 c, C는 不定
- A, a가 다른 상한이면 解 없음.

$a \neq b$: $\sin a < \sin A \sin b$ 이면 解 없음
$\sin a = \sin A \sin b$ 이면 $B = \frac{\pi}{2}$
$\sin a > \sin A \sin b$ 이면 解 2
$A-B, a-b$ 가 同符號이면, 그에 對한 解가 없음

3-3 公 式

① sin 法則 $\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}$ 에서 $\mu(a-b) - \mu(a+b) = \mu(A-B) - \mu(A+B)$

② $\cos\frac{a+b}{2} / \cos\frac{a-b}{2} = \tanh\frac{\mu(a)+\mu(b)}{2}$

③ $\sin\frac{a+b}{2} / \sin\frac{a-b}{2} = \coth\frac{\mu(b)-\mu(a)}{2}$

④ $\cos\frac{A+B}{2} / \mathrm{cns}\frac{A-B}{2} = \tanh\frac{\mu(A)+\mu(B)}{2}$

⑤ $\sin\frac{A+B}{2} / \sin\frac{A-B}{2} = \coth\frac{\mu(B)-\mu(A)}{2}$

Napier 의 比例式에서

① $\mu(a+b) - \mu(c) = -\log\ \coth\frac{\mu(A)+\mu(B)}{2}$

② $\mu(a-b) - \mu(c) = \log\ \coth\frac{\mu(B)-\mu(A)}{2}$

③ $\mu(A+B) + \mu(C) = -\log\ \coth\frac{\mu(a)+\mu(b)}{2}$

④ $\mu(A-B) + \mu(C) = \log\ \coth\frac{\mu(b)-\mu(a)}{2}$

3-4 $\cos A = -\cos B \cdot \cos C + \sin B \cdot \sin C \cdot \cos a$
$\cos B = -\cos C \cdot \cos A + \sin C \cdot \sin A \cdot \cos b$

$A+B<\pi$	$A<B$	$A>B$
$\dfrac{\sinh\left\{\frac{\mu(A+B)+\mu(A-B)}{2}+\mu(C)\right\}}{\sinh\frac{\mu(A-B)-\mu(A+B)}{2}}$	$= \begin{cases}\cos b \\ \tanh\ \mu(b)\end{cases}$	$= \begin{cases}\cos\ a \\ \tanh\ \mu(a)\end{cases}$
$\dfrac{\coth\left\{\frac{\mu(A+B)+\mu(A-B)}{2}+\mu(C)\right\}}{\cosh\frac{\mu(A-B)-\mu(A+B)}{2}}$	$= \begin{cases}\cos a \\ \tanh\ \mu(a)\end{cases}$	$\doteq \begin{cases}\cos\ b \\ \tanh\ \mu(b)\end{cases}$
$A+B>\pi$		
$-\dfrac{\cosh\left\{\frac{\mu(A+B)+\mu(A-B)}{2}-\mu(C)\right\}}{\cosh\frac{\mu(A-B)-\mu(A+B)}{2}}$	$= \begin{cases}\cos\ b \\ \tanh\ \mu(b)\end{cases}$	$= \begin{cases}\mathrm{coa}\ a \\ \tanh\ \mu(a)\end{cases}$

$-\dfrac{\sinh\left\{\dfrac{\mu(A+B)+\mu(A-B)}{2}-\mu(C)\right\}}{\sinh\dfrac{\mu(A-B)-\mu(A+B)}{2}}$	$=\begin{cases}\cos a\\ \tanh\mu(a)\end{cases}$	$=\begin{cases}\cos b\\ \tanh\mu(b)\end{cases}$
$\dfrac{\sinh\mu(A)}{e^{\mu(c)}}$	$A=B$	

以上과 같은 方法으로 $\mu(\theta)=\log\left|\cot\dfrac{\theta}{2}\right|$ 의 값 및 그의 双曲線函數의 값을 求하면 斜角球面三角形의 解法의 公式을 유도할 수 있다.

3—5. 公式을 利用한 球面三角形의 問題의 풀이

上述한 바와 같이 誘導된 여러가지 公式을 利用하여 實例를 들면

EX. 1. 三邊이주어질때

球面三角形에서 $a=70°14'20''$, $b=49°24'10''$, $c=38°46'10''$

일때 A, B, C, 를 求하라. (金相輪 球面三角法 133頁의 例題)

$\left.\begin{aligned}a&=70°14'20''\\ b&=49°24'10''\\ c&=38°46'10''\end{aligned}\right\}$에서 $a+b<\pi, a>b$, 이므로 10頁의 表에 依한

公式 $\dfrac{\sinh\mu\left\{\dfrac{\mu(a+b)+\mu(a-b)}{2}-\mu(c)\right\}}{\sinh\dfrac{\mu(a-b)-\mu(a+b)}{2}}=\begin{cases}\cos A\\ \tanh\mu(A)\end{cases}$ 을 利用한다.

또한 $\log_e x=\log_{10}x\cdot\log_e 10=\log_{10}x\cdot 2.30256$

(解) $(a-b)/2=10°25'5''$ $c/2=19°23'5''$

$(a+b)/2=59°49'15''$

※ $\mu(a-b)=\log_e\cot\dfrac{a-b}{2}=\log\cot 10°25'5''\cdot\log_e 10=0.73551\times\log_e 10$

※ $\mu(a+b)=\log\cot 59°49'15''\cdot\log_e 10=\bar{1}.76457\times\log_e 10$

※ $\mu(c)=\log\cot 19°23'5''\cdot\log_e 10=0.45362\times\log_e 10$

$\alpha=\dfrac{\mu(a+b)+\mu(a-b)}{2}=0.25004\times\log_e 10$

$\beta=\dfrac{\mu(a-b)-\mu(a+b)}{2}=0.48547\times\log_e 10=1.11782$

$\alpha-\mu(c)=(0.25004-0.45362)\times\log_e 10=-0.20358\times 2.30256=-0.46876$

※ 위의 公式의 左邊 $=\dfrac{\text{sing}\ \{\alpha-\mu(c)\}}{\sinh B}=\dfrac{-\sinh 0.46876}{\sinh 1.11782}=\dfrac{-0.48612}{1.36561}=-0.35597$

※ $\cos A=-0.35597$ $A=110°51'12''$

① 위의 計算中 ※印은 3개의 $\log\cot\theta$, 2個의 $\sinh x$, 1個의 $\cos y$ 의 값으로 各各 數表에서 찾아야 한다.

② 公式 1個만으로 A를 求할수 있다.

③ 兩邊에 10을 底로하는 對數를 取하여 計算 할수도 있다.

④ 双曲線函數의 對數表를 作成하면 위의 計算을 容易하게 할수있다.

또 B를 求하려면 $a+b<\pi$, $a>b$이므로 10頁의 表에 依한 公式

$$\frac{\cosh\left\{\frac{\mu(a+b)+\mu(a-b)}{2}-\mu(c)\right\}}{\cosh\frac{\mu(a-b)-\mu(a+b)}{2}}=\begin{cases}\cos B\\ \tanh\mu(B)\end{cases}$$ 를 利用한다.

(解) 左邊 $=\frac{\cosh\{\alpha-\mu(c)\}}{\cosh\beta}=\frac{\mathrm{cash}(-0.46876)}{\cosh 1.11782}=\frac{1.11190}{1.69260}=0.65692$

$\cos B=0.65692 \quad \therefore B=48°56'6''$

① A를 求할때의 公式과 거이 同一하다. (cosh 과 cos 이 相異할 뿐)

② A를 求할때의 計算한 $\mu(a-b)$, $\mu(a+b)$, $\alpha-\mu(c)$, β 等은 그대로 代入하게 된다.

같은 方法으로 C에 對한 公式을 써서 그 값을 計算할수도 있고 sin法則을 利用하여 求할수도 있다.

驗算式은 sin法則; $\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$

EX. 2. 三角이주어질때

球面三角形에서 $A=60°$, $B=135°$, $C=60°$일대 a, b, c, 를 求하라. (金相輪 球面三角法 137頁의 例題)

$A+B>\pi$, $A<B$이미로 11頁의 表에 依한 公式

$$-\frac{\cosh\left\{\frac{\mu(A+B)+\mu(A-B)}{2}+\mu(C)\right\}}{\cosh\frac{\mu(A-B)-\mu(A+B)}{2}}=\begin{cases}\cos b\\ \tanh(b)\end{cases}$$ 을 利用한다.

(解) ※ $\mu(A-B)=\log_{10}\cot 37°30'\cdot\log_e 10=0.11502\times\log_e 10$

※ $\mu(A+B)=\log\ \cot 97°30'\cdot\log_e 10=\bar{1}.11943\times\log_e 10$

※ $\mu(C)=\log\ \cot 30°\cdot\log_e 10=0.23856\times\log_e 10$

$\alpha=\frac{\mu(A+B)+\mu(A-B)}{2}=-0.38277\times\log_e 10$

$\beta=\frac{\mu(A-B)-\mu(A+B)}{2}=0.49780\times\log_e 10=1.14621$

$\alpha+\mu(C)=(-0.38277+0.23856)\cdot\log_e 10=-0.33184$

※ 위 公式의 左邊 $=-\frac{\cosh\ \{\alpha+\mu(C)\}}{\cosh\ \beta}=-\frac{1.05558}{1.72606}=0.61155$

※ $\cos b=-0.61155 \quad \therefore\ b=127°42'5''$

또 a를 求하기 爲하여 $A+B>\pi$, $A<B$이므로 11頁에 依한

公式; $$-\frac{\sinh\left\{\frac{\mu(A+B)+\mu(A-B)}{2}+\mu(C)\right\}}{\sinh\frac{\mu(A-B)-\mu(A+B)}{2}}=\begin{cases}\cos a\\ \tanh\mu(a)\end{cases}$$

위에서 b를 求할때의 計算結果를 利用하면

公式의 左邊 $=-\frac{\sin\{\alpha+\mu(C)\}}{\sinh\ \beta}=\frac{0.33794}{1.41422}=0.23896$

$\cos a=0.23896 \quad \therefore\ a=76°10'30''=C.$

驗算式은 sin 法則; $\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$

EX. 1, EX. 2를 比較綜合하면

$\left.\begin{matrix} a+b<\pi \\ a>b \end{matrix}\right\}$ 인때 A를 求하기 爲해서 公式 $\dfrac{\sinh\left\{\dfrac{\mu(a+b)+\mu(a-b)}{2}-\mu(c)\right\}}{\sinh\dfrac{\mu(a-b)-\mu(a+b)}{2}}=\left\{\begin{matrix}\cos A \\ \tanh A,\end{matrix}\right.$

$\left.\begin{matrix} a+b>\pi \\ A<B \end{matrix}\right\}$ 인때 a를 求하기 爲해서 公式 $-\dfrac{\sinh\left\{\dfrac{\mu(A+B)+\mu(A-B)}{2}+\mu(C)\right\}}{\sinh\dfrac{\mu(A-B)-\mu(A+B)}{2}}=\left\{\begin{matrix}\mathrm{sos}\,a \\ \tan\mu(a)\end{matrix}\right.$

인 公式을 適用하였다. 여기에서 公式의 型態가, a와 A, b와 B를 바꾼모양으로 거의 同一하다는 것을 알 수 있으며 其他 다른 경우에도 같다. 그러므로 從來의 解法에서 쓰이는 公式보다 容易하게 記憶할 수 있을 것이다.

EX. 3. 두邊과 그 夾角이 주어질 때

球面三角形에서 $a=57°56'53''$, $b=137°20'33''$,

$C=94°48'6''$일때, A, B, c를 求하라(球面三角法 140頁의 例題)

Napier의 比例式에서 유도되는 公式

$$\left.\begin{matrix}\mu(A+B)=-\log\coth\dfrac{\mu(a)+\mu(b)}{2}-\mu(C) \\ \mu(A-B)=\log\coth\dfrac{\mu(b)-\mu(a)}{2}-\mu(C)\end{matrix}\right\}\text{에서 } A, B\text{를 求할수 있다.}$$

단 $a<b$면 $A<B$.

$\dfrac{a+b}{2}>90°$면 $\dfrac{A+B}{2}>90°$, $\dfrac{a+b}{2}<90°$면 $\dfrac{A+B}{2}<90°$

또 $\dfrac{a-b}{2}<0$이면 $\dfrac{A-B}{2}<0$, $\dfrac{a-b}{2}>0$이면 $\dfrac{A-B}{2}>0$,

$\mu(-\theta)=(\theta)$ (2頁參照).

(解) 위의 公式에 a, b, C를 代入하면

※ $\mu(a)=\log\cot 28°58'26.5''.\log_e 10=0.25671\times\log_e 10$

※ $\mu(b)=\log\cot 68°40'16.5''\cdot\log_e 10=\bar{1}.59160\times\log_e 10$

$\mu(c)=\log\cot 47°24'3''\cdot\log_e 10=\bar{1}.96356\times\log_e 10$

$\dfrac{\mu(a)+\mu(b)}{2}=\bar{1}.92416\times\log_e 10=-0.17463$

$\dfrac{\mu(b)-\mu(a)}{2}=-0.33256\times\log_e 10=-0.76574$

$\log\coth\dfrac{\mu(a)+\mu(b)}{2}=\log_{10}\coth(-0.17463)\cdot\log_e 10$

$=-\log 0.17287\times\log_e 10=-(\bar{1}.23772)\times\log_e 10$

$\log\coth\dfrac{\mu(a)-\mu(b)}{2}=\log_{10}\coth 0.76574\cdot\log_e 10$

$=-\log 0.64444\times\log_e 10=-(\bar{1}.80918)\times\log_e 10$

$\log\coth\dfrac{\mu(a)+\mu(b)}{2}+\mu(C)=0.72584\times\log_e 10$

$\therefore$ 公式 $\mu(A+B)=-\left\{\log\coth\dfrac{\mu(a)+\mu(b)}{2}+\mu(C)\right\}$에서

$$\mu(A+B)=\log\cot\frac{A+B}{2}=\log_{10}\cot\frac{A+B}{2}\cdot\log_e 10$$

$$\log_{10}\cot\frac{A+B}{2}=-0.72584=\bar{1}.27416$$

$$\frac{A+B}{2}\fallingdotseq 100°39' \quad \left(\frac{a+b}{2}>90 \quad \therefore \frac{A+B}{2}>90\right) \cdots\cdots ①$$

또 $\log\coth\frac{\mu(b)-\mu(a)}{2}-\mu(C)=-(\bar{1}.77353)\times\log_e 10$

$A<B, \ \mu(-\theta)=\mu(\theta)$

公式 $\mu(B-A)=\log\coth\frac{\mu(b)-\mu(a)}{2}-\mu(C)$

$$\log_e\cot\frac{B-A}{2}=-(\bar{1}.77353)\times\log_e 10$$

$$\log_{10}\cot\frac{B-A}{2}=0.22647$$

$$\frac{B-A}{2}\fallingdotseq 30°39'4'' \left(\frac{b-a}{2}<90, \ \therefore \frac{B-A}{2}<90\right)\cdots\cdots ②$$

①과 ②에서 $\begin{cases}A=69°59'56''\\B=131°18'4''\end{cases}$

또 c를 求하려면 Napier의 比例式에서 유도되는 公式(11頁)

$$\begin{cases}\log\coth\frac{\mu(A)+\mu(B)}{2}=-\mu(a+b)+\mu(c)\\ \text{或은 } \log\coth\frac{\mu(B)-\mu(A)}{2}=\mu(a-b)-\mu(c)\end{cases}$$

또는 sine法則 $\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$

또는 $-\dfrac{\sinh\left\{\frac{\mu(a+b)+\mu(a-b)}{2}-\mu(c)\right\}}{\sinh\frac{\mu(a-b)-\mu(a+b)}{2}}=\cos A \quad \left(\begin{cases}a+b\pi\\a<b\end{cases} \text{ 때}\right)$

또는 Gauss의 方程式 $\cos\frac{c}{2}=\dfrac{\cos\frac{a-b}{2}\cos\frac{C}{2}}{\sin\frac{A+B}{2}}, \quad \cos\frac{c}{2}=\dfrac{\cos\frac{a+b}{2}\sin\frac{C}{2}}{\cos\frac{A+B}{2}}$

에서 求할수 있다.

EX. 4. 두角 A. B와 그 夾邊 c가 주어질때

$$\left.\begin{aligned}\mu(a+b)&=-\log\frac{\mu(A)+\mu(B)}{2}+\mu(c)\\ \mu(a-b)&=\log\frac{\mu(B)-\mu(A)}{2}+\mu(c)\end{aligned}\right\}$$ 인 連立方程式에서 a, b를 求하고

Gauss의 方程式 $\cos\frac{C}{2}=\dfrac{\sin\frac{A+B}{2}}{\cos\frac{a-b}{2}}\cos\frac{c}{2}$

또는 $\mu(A+B)=-\log\coth\frac{\mu(a)+\mu(b)}{2}-\mu(C)$

或은 $\mu(A-B)=\log\coth\frac{\mu(b)-\mu(a)}{2}-\mu(C)$

에서 C를 求할수 있다.

이때 $\mu(-\theta)=\mu(\theta)$

$\frac{A+B}{2}>90°$이면 $\frac{a+b}{2}>90°$, $\frac{A-B}{2}<90°$이면 $\frac{a-b}{2}<90°$

또 $\frac{A-B}{2}<0$이면 $\frac{a-b}{2}<0$, $\frac{A-B}{2}>0$이면 $\frac{a-b}{2}>0$.

驗算式은 $\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$

EX. 5. 두邊 a, b와 그 한對角 A가 주어질 때

sin 法則에 依하여 $\sin B=\frac{\sin b}{\sin a}\sin A$ 에서 B는 2個求해짐.

그 各各에 對하여 Napier의 比例式에서 유도되는 4個의 公式中 어느 것을 適用하여도 된다.

吟味 ① $a=b$ 때 $(A=B)$

{ a, A가 同種의 角이면 解는 하나
{ a, A가 異種의 角이면 不能

② $\sin b\cdot\sin A>\sin a$이면 不能

③ $\sin b\cdot\sin A=\sin a$ 될 때

{ a. A가 同種의 角이면 解는 하나(이때 $B=90°$가됨)
{ a. A가 異種의 觀이면 不能

④ $\sin b\cdot \sin A<\sin a$ 될 때

sine 法則을 利用하여 求한 두개의 B가 $B_1, B_2=180-B_1$ 일때 $A-B_1$, $A-B_2$가 $a-b$와 同符號이면 B_1, B_2를 取하고 이때 解는 2個.

B_1, B_2 中 $A-B$가 $a-b$와 同符號가 아닌 것이 있으면 그것은 버린다. 이때는 解는 2個.

驗算

EX. 6. 두角 A, B와 그한 對邊 a가 주어질 때

sine 法則에 依하여 $\sin b=\frac{\sin B\ \sin a}{\sin\ A}$에서 b는 2個求해짐. 그 各各에 對하여 Napier 의 比例式에서 유도되는 4個의 公式中 어느 것을 適用하여도 된다. 求한 b의 값을 b_1, $b_2=180-b_1$이라 하자.

吟味 ① $\sin B\cdot\sin a>\sin A$ 이면 解는 없음. 即不能

② $\sin B\cdot\sin a=\sin A$ 이면 $b=90°$가 되고 解는 하나

③ $\sin B\cdot\sin a<\sin A$ 이면

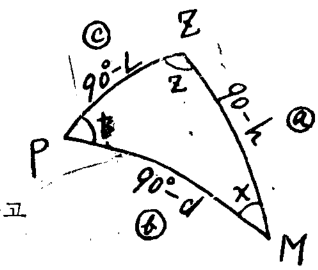
(i) $a-b_1$, $a-b_2$가 $A-B$와 同符號이면 b_1, b_2를 다 取하여 解는 2個

(ii) $a-b_1, a-b_2$ 中 $A-B$와 同符號인 것이 하나이면 그 b만 取하여 解는하나

(iii) $a-b_1$, $a-b_2$ 中 $A-B$와 同符號인 것이 없으면 不能

驗算式은 $\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$

4. 天文三角形에 對한 應用



$$\begin{cases}\sin h=\sin d\cdot\sin L+\cos d\cdot\cos L\cdot\cos t\\ \sin d=\sin h\cdot\sin L+\cos h\cdot\cos L\cdot\cos z\\ \sin L=\sin d\cdot\sin h+\cos d\cdot\cos h\cdot\cos X\end{cases}$$

$90°-h=a$, $90°-d=b$, $90°-L=c$를 10 頁의 表에 代入하고

$\mu(a-b)=\mu(h-d)$, $\mu(a+b)=-\mu(h+d)$를 利用하면

$h+d>0$	$h>d$	$h<d$
$\sinh\left\{\frac{\mu(h-d)-\mu(h+d)}{2}-\lambda(L)\right\}\Big/\sinh\frac{\mu(h-d)+\mu(h+d)}{2}$	$=\begin{cases}\cos z\\ \tanh\mu(z)\end{cases}$	$=\begin{cases}\cos t\\ \tanh\mu(t)\end{cases}$
$\cosh\left\{\frac{\mu(h-d)-\mu(h+d)}{2}-\lambda(L)\right\}\Big/\cosh\frac{\mu(h-d)+\mu(h+d)}{2}$	$=\begin{cases}\cos t\\ \tanh\mu(t)\end{cases}$	$=\begin{cases}\cos z\\ \tanh\mu(z)\end{cases}$
$d+L>0$	$d>L$	$d<L$
$\sinh\left\{\frac{\mu(d-L)-\mu(d+L)}{2}-\lambda(h)\right\}\Big/\sinh\frac{\mu(d-L)+\mu(d+L)}{2}$	$=\begin{cases}\cos X\\ \tanh\mu(X)\end{cases}$	$=\begin{cases}\text{css } z\\ \tanh\mu(z)\end{cases}$
$\cosh\left\{\frac{\mu(d-L)-\mu(d+L)}{2}-\lambda(h)\right\}\Big/\cosh\frac{\mu(d-L)+\mu(d+L)}{2}$	$=\begin{cases}\cos z\\ \tanh\mu(z)\end{cases}$	$=\begin{cases}\cos X\\ \tanh\mu(X)\end{cases}$
$L+h>0$	$L>h$	$L<h$
$\sinh\left\{\frac{\mu(L-h)-\mu(L+h)}{2}-\lambda(d)\right\}\Big/\sinh\frac{\mu(L-h)+\mu(L+h)}{2}$	$=\begin{cases}\cos t\\ \tanh\mu(t)\end{cases}$	$=\begin{cases}\cos X\\ \tanh\mu(X)\end{cases}$
$\cosh\left\{\frac{\mu(L-h)-\mu(L+h)}{2}-\lambda(d)\right\}\Big/\cosh\frac{\mu(L-h)+\mu(L+h)}{2}$	$=\begin{cases}\cos X\\ \tanh\mu(X)\end{cases}$	$=\begin{cases}\cos t\\ \tanh\mu(t)\end{cases}$

以上과 같이 天文三角形에서 觀則者의 緯度 L, 天體의 時角 t, 天體의 赤緯 d, 天體의 高度 h 天體의 方位 z 中 어느 3 個가 주어지면 天文三角形을 풀이하므로써 나머지 것을 求할 수 있다.

EX. 1. Given altitude (h), declination (d), and latitude (L), to find meridian angle (t) and azimuth angle (z).

兩邊에 對數를 取해서 計算할 수도 있다.

$h+d>0$	$h>d$	$h<d$
$\log\sinh\left\{\frac{\mu(h-d)-\mu(h+d)}{2}-\lambda(L)\right\}-\log\sinh\frac{\mu(h-d)+\mu(h+d)}{2}$	$=\begin{cases}\log\cos z\\ \log\tanh\mu(z)\end{cases}$	$=\begin{cases}\log\cos t\\ \log\tanh\mu(t)\end{cases}$
$\log\cosh\left\{\frac{\mu(h-d)-\mu(h+d)}{2}-\lambda(L)\right\}-\log\cosh\frac{\mu(h-d)+\mu(h+d)}{2}$	$=\begin{cases}\log\cos t\\ \log\tanh\mu(t)\end{cases}$	$=\begin{cases}\log\cos Z\\ \log\tanh\mu(z)\end{cases}$

$$\begin{cases} \mu(\alpha)=\dfrac{\mu(h-d)+\mu(h+d)}{2} \\ \mu(\beta)=\dfrac{\mu(h-d)-\mu(h+d)}{2} \end{cases}$$ 라놓면

1	L			6	$\lambda(L)$
2	h	4	$h-d=a$	7	$\mu(h-d)=\mu(a)$
3	d	5	$h+d=b$	8	$\mu(h+d)=\mu(b)$
				9	$\mu(\beta)=\dfrac{\mu(a)-\mu(b)}{2}$
12	log ccosh $\mu(\alpha)$	13	log sinh $\mu(\alpha)$	10	$\mu(\alpha)=\dfrac{\mu(a)+\mu(b)}{2}$
14	log cosh$\{\mu(\beta)-\lambda(L)\}$	15	log sinh$\{\mu(\beta)-\lambda(L)\}$	11	$\mu(\beta)-\lambda(L)$
16	log tanh $\mu(A)$ $=12-14$	17	log tanh $\mu(B)$ $=13-15$	20	z
18	A	19	B	21	t

$h>d: t=A,\ z=B,\ h<d: t=B\ z=A$

$$\begin{cases} \log\tanh u(A)=\log\cosh\{u(\beta)-\lambda(L)\}-\log\cosh\mu(\alpha) \\ \log\tanh u(B)=\log\sinh\{u(\beta)-\lambda(L)\}-\log\sinh\mu(\alpha) \end{cases}$$

6. 結　論

지금까지 結果를 要約하면 다음과 같다.

① $e^{u(\theta)}=\left|\cot\frac{\theta}{2}\right|$ 즉 $u(\theta)=\log_e\left|\cot\frac{\theta}{2}\right|$인 函數를 定義하고

② θ의 円函數를 $\mu(\theta)$ 및 $\lambda(\theta)$의 双曲線函數로 유도하고

③ $u(\theta)$의 双曲線函數를 利用해서 斜角球面三角形의 解法의 公式을 유도하고

④ 天文三角形에 對한 應用을 略述하였다.

$\mu(\theta)=\log_e\left|\cot\frac{\theta}{2}\right|$ 와 $\lambda(\theta)=\log_e\left|\cot\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)\right|$ 의 双曲線函數와 그 應用에 關하여

$u(\theta)=\log_e\left|\cot\frac{\theta}{2}\right|$의 값, 또 이 값에 對한 双曲線函數表를 作成하면 斜角球面三角形 및 天文三角形의 풀이를 容易하게 할 수 있다.

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株式會社의 理事會에 對한 諸問題

全 秉 翼

Problem concerning Board of Directors of Jointstock Corporation

Abstract

It seems true that tho development of captalistic economy has made a great deal of varation in both the type and substance of Joint-stock corporation.

Nowdays, the higer standard of captalistic economy and the seperation between ownership and mangement has necessorily required mobility along with nationality on the side of those who run the bussiness body.

On the demand of mobility and efficiency in the present business, Board of Directors has beeome to occupy on important position inthe organigation of Joint-stock corporation.

However, It is also true that the legal regulation for the system of Board of Directors and its operation have shown great focus theoretically and practically. So in order to make more efficientoperation of the system of Board of Directors, This essay will historically explain the consequnt result that the Board of Directors should appear, and preventing it from reducing itself incompetent, the substance of law and more efficient operation Board of Directors are to be discoverd through explaning the problems found on the exsisting law from the view-point of the legislative theory.

<目 次>

1. 序 論

資本主義經濟의 發展이 株式會社인 企業의 形態 및 實質에 많은 變容를 招來하였다. 近代에 있어서의 資本主義經濟의 高度化가 株式會社企業의 中樞的 經營者層 即 管理機關에 機動的이고 効率的인 活動을 要請함에 따라 그의 構成 및 權限의 配分에 關한 再檢討가 近代株式會社의 法에 焦點이 되고 있다.

近代株式會社의 經營機構는 意思機關과 管理機關으로 分離하고, 意思機關은 會社資本의 形成에 參加한 사람들을 構成員으로 하는 最上級機關으로서 管理機關을 그 支配下의 下級機關으로