Low Sensitive Optimal Steering System of Ships at Sea

Part 1. Optimal control system with consideration of sensitivity when some of variables are not measurable

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⟨contents⟩

Abstract

制御系의 設計時에 사용하는 파라메터는 制御系의 모델을 구성할 때의 간략화 혹은 측정시의 오차 등으로 인하여 실제의 값과의 사이에는 많은 誤差가 豫想되며, 특히 선박의 경우와 같이 파라메터인 시정수(時定數)가 선박의 적화 및 항해상태에 따라 변화하게 되는 경우에는, 지금까지 사용되어 온 방법에 의한 최적설계(最適設計)는 거의 무의미하게 된다. 본논문의 제1部에서는 설계시에 발생하는 이러한 문제점을 해결하기 위하여 觀測器를 도입하여 제어계를 설계할 경우에 파라메터의 변화에 대응할 수 있는 확장된 의미에서의 평가함수를 제안하고, 그 평가함수의 성질을 논함과 동시에 평가함수의 결정에 필요한 충분조건을 얻었다. 본 논문에서 제안한 평가함수는 觀測器의 능력과 밀접한 관계가 있으며, 제어계의 성능을 평가하는 지침으로서 그 응용범위가 매우 넓을 것으로 생각된다.

1. Introduction

In optimal regulator problems an increase in the value of the performance index is in general encountered when a Luenberger type obserever is employed to construct the estimates of the state variables which are not available by direct measurement, and in almost cases the increase in the performance index becomes large when the real parts of the observer eigenvalues become highly negative (J. J. Bongiorno and D. C. Youla, 1968).

While, in the the case that the system parameters vary the cost varies, i.e., for a different set of parameter values the cost and optimal control have different values and the associated transient response has a different characteristics.

In fact, system parameters often differ from their nominal values due to uncertainties and errors in measurement or simplifications in mathematical modelling, and change by the change of the loading condition as is usually in the case of the ship.

Especially under the circumstances which impossible to measure the system parameters every time its vary, it would be desirable to construct optimal law as low sensitive as possible over a range of partmeter variations.

Bongiorno (1973) showed that designing observer for insensitivity to system parameter variation is impossible for all initial states, but low sensitive controller which is constructed from the estimated state variables is rather demanding.

In this paper, for the purpose of constructing comparatively low sensitive control law a general index of optimality which includes both sensitivity and performance characteristics are introduced. Moreover it is discussed for what performance index the control law using the estimates of the state variables is optimal, and a sufficient condition is derived.

The system under consideration is one-input and one-output completely controllable and observable linear time invariant dynamical system as,

$$\left.\begin{aligned} \dot{X}&=AX+bu \\ y&=c'X \end{aligned}\right\} \quad \cdots\cdots(1.1)$$

where X is an n×1 state vector, A, b, and c are n×n, n×1, n×1 matrices respectively, and can be easily expressed by an observable canoic form with,

$$A=\begin{pmatrix} 0 & \cdots\cdots & -a_0 \\ 1 & 0 & -a_1 \\ \cdot & & \vdots \\ 0 & 1 \cdot & \vdots \\ & 1 & -a_{n-1} \end{pmatrix} \qquad b=\begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ \vdots \\ b_{n-1} \end{pmatrix} \qquad c=\begin{pmatrix} 0 \\ \\ \\ 1 \end{pmatrix}$$

The performance index is chosen to be quadratic form as:

$$J=1/2\int_0^\infty (X'QX+ru^2)dt \quad \cdots\cdots(1.2)$$

where Q is as n×n positive definite symmetric cost weighting matrix, and r is a scalar.

The optimal control is given by,

$$u=-KX \quad \cdots\cdots(1.3)$$

and K is an 1×n gain matrix defined by

$$K=rb'P$$

and P is the positive definite n-square matrix solution of the Riccati equation:

$$A'P+PA+Q-\mathrm{Pbrb}'P=0 \quad \cdots\cdots(1.4)$$

The optimal value of this performance index is :

$$J=1/2X'(0)PX(0) \quad \cdots\cdots(1.5)$$

2. Observation of the state vector

The observation problem is the reconstruction of the state vector for the system given by eq. (1.1) from the available output and the control.

In the case of n-dimensional, Luenberger type observer is defined as (Anderson et al., 1971),

$$\dot{Z}=AZ+bu+f(y-c'Z)$$
$$=(A-fc')Z+bu+fy \cdots\cdots (2.1)$$

with arbitrary eigenvalues and

$$Z(t)=X(t)+\exp(A-fc')t\cdot e(0) \cdots\cdots (2.2)$$

where Z and f are $n\times 1$ vector.

On the other hand, n−1 dimensional observer is:

$$\dot{w}=Ew+du+gy \cdots\cdots (2.3)$$

where

$$E=\begin{pmatrix} 0 & \cdots & & -\alpha_0 \\ 1 & & 0 & -\alpha_1 \\ & \cdot & & \vdots \\ & & 1 & \vdots \\ & & \cdot & \vdots \\ 0 & & 1 & -\alpha_{n-1} \end{pmatrix} \qquad d=\begin{pmatrix} b_0-\alpha_0 b_{n-1} \\ \vdots \\ b_{n-2}-\alpha_{n-2}b_{n-1} \end{pmatrix}$$

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$$g=\begin{pmatrix} 0 \\ \alpha_0 \\ \vdots \\ \alpha_{n-3} \end{pmatrix}-\begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-2} \end{pmatrix}-\begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{n-2} \end{pmatrix}(\alpha_{n-2}-a_{n-1})$$

where w is an n−1 vector and α_i $(i=0, 1, \cdots\cdots, n)$ are arbitrary eigenvalues of the observer. For the dynamic system to be an observer the following relationship must hold :

$$w=TX+e \cdots\cdots (2.4)$$
$$\dot{e}=Ee \cdots\cdots (2.5)$$

with

$$T=\begin{pmatrix} 1 & & & -\alpha_0 \\ & \cdot & 0 & -\alpha_1 \\ & 1 & & \vdots \\ 0 & & \cdot & \vdots \\ & & 1 & -\alpha_{n-2} \end{pmatrix}$$

The n vector $\tilde{X}$ is an estimate of the system state X and is :

$$\tilde{X}=nw+my \cdots\cdots (2.6)$$

or

$$\tilde{X}=X+ne \cdots\cdots (2.7)$$

where

$$n=\begin{pmatrix} I_{n-1} \\ \cdots\cdots \\ 0 \end{pmatrix} \qquad m=\begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{n-2} \\ 1 \end{pmatrix}$$

If the state is measured indirectly by the observer, an increase in the performance index of eq. (1.2) is expected. In this case, to obtain the optimal control ignoring the observer charactersitics will be meaningless.

The same statments are true for the case that the system parameters are varied.

Consider here the system equations for the variation of system parameters.

It is assumed that the matrices A, b in eq. (1.1) and W, d, g in eq. (2.3) are function of a system parameter τ.

Differentiating eq. (1.1) with respect to τ yields :

$$\left.\begin{aligned} \dot{X}_\tau&=AX_\tau+A_\tau X+bu_\tau+b_\tau u, \qquad X(t_0)=X_\tau(0) \\ y_\tau&=c'X_\tau \end{aligned}\right\} \quad \cdots\cdots(2.8)$$

and with the same way, the following equations are obtained from the eq. (2.3) and (2.4) :

$$\dot{W}_\tau=EW_\tau+du_\tau+d_\tau u+g_\tau y+gy_\tau \quad \cdots\cdots(2.9)$$

$$\dot{W}_\tau=TX_\tau+e_\tau \quad \cdots\cdots(2.10)$$

Substituting eq. (2.10) into eq. (2.9) and utilizing eqs. (1.1) and (2.8) yields :

$$(TA-ET-gc')X_\tau+(Tb_\tau-d_\tau)u+(Tb-d)u_\tau+(TA_\tau-g_\tau c')X+\dot{e}_\tau=Ee_\tau$$

or

$$\dot{e}_\tau=Ee_\tau \quad \cdots\cdots(2.11)$$

The subscript indicates the partial derivative with respect to τ such as $X_\tau=\partial X/\partial\tau$.

3. Augumented performance index

As the estimates of state variables $\tilde{X}$ can be represented as a linear function of the state variables X and the estimated error e, the following system equations including an observer characteristics are obtained from the eqs. (1.1) and (2.5) :

$$\left.\begin{aligned} \dot{\tilde{X}}&=\tilde{A}\tilde{X}+\tilde{b} \\ y&=\tilde{c}'\tilde{X} \end{aligned}\right\} \quad \cdots\cdots(3.1)$$

where $\tilde{A}$ and $\tilde{b}$ are $(2n-1)\times(2n-1)$ and $(2n-1)\times l$ matrices respectively, and $\tilde{X}$ is an $(2n-1)$ dimensional vector with

$$\tilde{A}=\begin{bmatrix} A & \vdots & 0 \\ 0 & \vdots & E \end{bmatrix} \qquad \tilde{b}=\begin{bmatrix} b \\ \cdots \\ 0 \end{bmatrix} \qquad \tilde{c}=\begin{bmatrix} c \\ \cdots \\ 0 \end{bmatrix}$$

and $$\tilde{X}=(X \vdots e)' \quad \cdots\cdots(3.2)$$

Also, as the performance index corresponds to the system equations (3.1) the following is assumed:

$$\tilde{J}=1/2\int_0^\infty(\tilde{X}'\tilde{Q}\tilde{X}+ru^2)dt \quad \cdots\cdots(3.3)$$

and it is assumed that the structure of the weighting matrix is unknown.

In similar way, the following system equations for the system parameter variation are obtained from the eqs. (2.8) and (2.11):

$$\left.\begin{aligned} \dot{X}_\tau &= AX_\tau + A_\tau X + du_\tau + b_\tau u \\ e_\tau &= Ee_\tau \\ y_\tau &= c'X_\tau \end{aligned}\right\} \quad \cdots\cdots (3.4)$$

Combining eqs. (3.2) and (3.4), the augmented system equations are obtained such that,

$$\left.\begin{aligned} \dot{\underline{X}} &= \underline{A}\underline{X} + \underline{B}\underline{u}, \qquad \underline{X}(t_0) = \underline{X}(0) \\ \underline{y} &= \underline{C}\underline{X} \end{aligned}\right\} \quad \cdots\cdots (3.5)$$

where $\underline{X}$ and $\underline{X}(0)$ are (4n−2) dimensional vectors and $\underline{u}$ and $\underline{y}$ 2-dimensional vectors defined as:

$$\left.\begin{aligned} \underline{X} &= [\tilde{X}, \tilde{X}_\tau]', \qquad \underline{X}(0) = [\tilde{X}(0), \tilde{X}_\tau(0)]' \\ \underline{u} &= [u, u_\tau]', \qquad \underline{y} = [y, y_\tau]' \end{aligned}\right\} \quad \cdots\cdots (3.5a)$$

and $\underline{A}, \underline{B}$ and $\underline{C}$ are respectively (4n−2)×(4n−2), (4n−2)×2 and 2×(4n−2) matrices defined below:

$$\underline{A} = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{A}_\tau & \tilde{A} \end{bmatrix}, \qquad \underline{B} = \begin{bmatrix} \tilde{b} & 0 \\ \tilde{b}_\tau & \tilde{b} \end{bmatrix}, \qquad \underline{C} = \begin{bmatrix} \tilde{c} & 0 \\ 0 & \tilde{c} \end{bmatrix} \quad \cdots\cdots (3.5b)$$

The vectors $\tilde{X}_\tau, u_\tau$ and y_τ are called the trajectory sensitivity vector, the control sensitivity vector and the output sensitivity vector respectively.

On the other hand, if the control u depends on a parameter τ, the cost function of eq. (3.3) will be a function of τ.

Here, the performance index sensitivity is defined as:

$$\tilde{J}_\tau = 1/2\int_0^\infty [(X_\tau'Q'X + X'QX_\tau) + (e_\tau'Q'_b e + e'Q_b e_\tau) + (e_\tau'Q'_{14}X + X'0_{14}e_\tau) + (X_\tau'Q_{14}e + e'Q_{14}X_\tau) + (u_\tau'ru + u'ru_\tau)]dt \quad \cdots\cdots (3.6)$$

$\tilde{J}_\tau$ represents a measure of the sensitivity of $\tilde{J}$ with respect to variation of the parameter τ. Since the system trajectory also depends on τ, it is reasonable to introduce into another sensitivity function as trajectory sensitivity defined by:

$$\tilde{J}_{t\tau} = 1/2\int_0^\infty [(X_\tau'QX_\tau + e_\tau Q_c e_\tau) + (e_\tau'Q_{34}'X_\tau + X_\tau'Q_{34}e_\tau) + u_\tau'ru_\tau]dt \quad \cdots\cdots (3.7)$$

In order to include sensitivity in a general index, the usual performance index is augumented to inclue both performance index itself and trajectory sensitivity function.

As the output is a linear function of the estimates of the state variables, the augumented performance index is taken to be,

$$\begin{aligned} \underline{J} &= J + \rho\tilde{J}_\tau + \beta^2\tilde{J}_{t\tau} + (e'Q'_{12}X + X'Q_{12}e) \\ &= 1/2\int_0^\infty (\underline{X}'\underline{Q}\underline{X} + \underline{u}'\underline{R}\underline{u})dt \quad \cdots\cdots (3.8) \end{aligned}$$

where

$$\underline{Q} = \begin{bmatrix} Q_1 & Q_2 \\ Q_2' & Q_3 \end{bmatrix}\begin{matrix} \}(2n-1) \\ \}(2n-1) \end{matrix} \qquad R = \begin{bmatrix} r & \rho r \\ \rho r & \beta^2 r \end{bmatrix} \quad \cdots\cdots (3.8a)$$

with

$$Q_1 = \begin{bmatrix} Q & Q_{12} \\ Q'_{12} & Q_a \end{bmatrix} \qquad Q_2 = \begin{bmatrix} \rho Q & \rho Q_{14} \\ \rho Q'_{14} & \rho Q_b \end{bmatrix} \qquad Q_3 = \begin{bmatrix} \beta^2 Q & \beta^2 Q_{34} \\ \beta^2 Q'_{34} & \beta^2 Q_c \end{bmatrix}$$

The weighting coefficients ρ and β permit the sensitivity functions to receive different emphasis. It is easy to see that R is postive definite if and only if the weighting constants ρ and β satisfy the inequality such as $\beta^2 > \rho^2$.

Appling the conventional optimal regulator theory to the generalized eqs. (3.5) and (3.8), an optimal control law is obtained as :

$$\underline{u}=-\underline{F}\underline{X}$$
$$=-R^{-1}\underline{B}'\underline{P}\underline{X} \qquad \cdots\cdots(3.9)$$

where $\underline{P}$ is the positive definite (4n−2)-square matrix solution of the Riccati equation,

$$\underline{A}'\underline{P}+\underline{P}\underline{A}+\underline{Q}=\underline{P}\underline{B}R^{-1}\underline{B}'\underline{P} \qquad \cdots\cdots(3.10)$$

and the optimal value of the augumented performance index is given by,

$$\underline{J}=1/2\underline{X}(0)\underline{P}\underline{X}(0) \qquad \cdots\cdots(3.11)$$

Let us consider before proceeding a few point about the solution for the augumented performance index $\underline{J}$.

As the pair (A, b) is completely controllable, the positive definite P_f is obtained from the eq. (A-5) in Appendix and it is well known that $A_f-b_fR^{-1}b'P_f$ has eigenvalues with negative real parts. Also, it is obvious that P_f is the solution of the Riccati equation when the state variables can be directly measured. Simlarly, since E is an stable matrix, the solution of eq. (A-6) is present.

From the above mentioned facts, it it easily known that the soution of eq. (3.10) are asymptotically stable and $\underline{J}$ is finite.

The optimal control of eq. (3.9) is rewritten for the implementation practical system as :

$$\begin{pmatrix} u \\ u_\tau \end{pmatrix} = -\begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}\begin{pmatrix} \tilde{X} \\ \tilde{X}_\tau \end{pmatrix} \qquad \cdots\cdots(3.12)$$

Using the above equation, the closed loop system equations become,

$$\left.\begin{aligned} &\dot{\tilde{X}}=(\tilde{A}-\tilde{b}\tilde{F}_{11})\tilde{X}_\tau-\tilde{b}F_{12}\tilde{X}, && \tilde{X}(t_0)=\tilde{X}(0) \\ &\dot{\tilde{X}}_\tau=(\tilde{A}-\tilde{b}_\tau F_{12}-\tilde{b}F_{22})\tilde{X}_\tau+(\tilde{A}_\tau-\tilde{b}_\tau F_{11}-\tilde{b}F_{12})\tilde{X}, && \tilde{X}_\tau(t_0)=\tilde{X}_\tau(0) \end{aligned}\right\} \cdots(3.13)$$

4. Some properties of the augumented performance index

From the facts mentioned above, a natural question which arises is whether the control law of eq. (3.9) can be exactly constructed from the estimates of the state variables.

In this section, one consider some sufficient conditions for which the weighting matrix Q must satisfy.

From the eqs. (2.6) and (2.7), the estimates of the state variables are obtained as :

$$\left.\begin{aligned} &\tilde{X}=X+ne \\ &\tilde{X}_\tau=X_\tau+ne_\tau \end{aligned}\right\} \qquad \cdots\cdots(4.1)$$

or

$$\left.\begin{aligned} &\tilde{X}=nW+my \\ &\tilde{X}_\tau=nW_\tau+my_\tau \end{aligned}\right\} \qquad \cdots\cdots(4.1a)$$

Substituting eq. (4.1) into eq. (3.12), the control law is rewritten as :

$$\begin{pmatrix} u \\ u_\tau \end{pmatrix}=-\begin{pmatrix} F_{11}X+F_{11}ne+F_{12}X_\tau+F_{12}ne_\tau \\ F_{21}X+F_{21}ne+F_{22}X_\tau+F_{22}ne_\tau \end{pmatrix} \qquad \cdots\cdots(4.2)$$

On the other side, combining the eqs. (3.5a) and ((3.8a), the eq. (3.9) expressed as the another form as :

$$\begin{pmatrix} u \\ u_\tau \end{pmatrix} = -\frac{1}{\xi}\begin{pmatrix} \beta^2{}_r\widetilde{b}' & \beta^2{}_r\widetilde{b}'_\tau - \rho r\widetilde{b}' \\ -\rho r\widetilde{b}' & -\rho r\widetilde{b}_\tau{}' + r\widetilde{b}' \end{pmatrix}\begin{pmatrix} P_1 & P_2 \\ P_2' & P_3 \end{pmatrix}\begin{pmatrix} \widetilde{X} \\ \widetilde{X}_\tau \end{pmatrix}$$

$$= -\begin{bmatrix} F_{11}X+F_a e+F_{12}X_\tau+F_b e_\tau \\ F_{21}X+F_c e+F_{22}X_\tau+F_d e_\tau \end{bmatrix} \quad \cdots\cdots (4.3)$$

where $\xi = r^2(\beta^2-\rho^2)$ and $\underline{P}$ is partitioned as,

$$P=\begin{bmatrix} \overbrace{P_1}^{2n-1} & \overbrace{P_2}^{2n-1} \\ P'_2 & P_3 \end{bmatrix},\quad P_1=\begin{bmatrix} \overbrace{M_1}^{n} & \overbrace{M_2}^{n-1} \\ M'_2 & M_3 \end{bmatrix},$$

$$P_2=\begin{bmatrix} L_1 & L_1 \\ L'_2 & L_3 \end{bmatrix},\quad P_3=\begin{bmatrix} 0_1 & 0_2 \\ 0'_2 & 0_3 \end{bmatrix} \quad \cdots\cdots (4.3a)$$

and

$$F_{11}=(\beta^2 rb'N_1+B^2 rb'_\tau L'_1-\rho rb'L'_1)/\xi,\quad F_a=(\beta^2 rb'M_2+B^2 rb'_\tau L_2-\rho rb'L_2)/\xi$$

$$F_{12}=(\beta^2 rb'L_1+\beta^2 rb'_\tau 0_1-\rho rb'0_1)/\xi,\quad F_b=(\beta^2 rb'L_2+\beta^2 rb'_\tau 0_2-\rho rb'0_2)/\xi$$

$$F_{21}=(-\rho rb'M_1-\rho rb'_\tau L_1-rb'L'_1)/\xi,\quad F_c=(-\rho rb'M_2-\rho rb_{,\tau}L_2-rb'L_2)/\xi$$

$$F_{22}=(-\rho rb'L_1-\rho rb'_\tau 0_1-rb'0_1)/\xi,\quad F_d=(-\rho rb'L_2-\rho rb'_\tau 0_2-rb'0_2)/\xi$$

In order to be expressed the control law as the state variables, eq. (4.2) must be agreed to eq. (4.3) precisely, i.e.,

$$F_a=F_{11}n$$
$$F_b=F_{12}n$$
$$F_c=F_{21}n$$
$$F_d=F_{22}n \quad \cdots\cdots (4.4)$$

and from the eq. (4.4) the following sufficient conditions are naturally derived.

$$M_2=M_1 n$$
$$L_2=L'_1 n$$
$$0_2=0_1 n \quad \cdots\cdots (4.5)$$

Also it is not difficult to obtain the necessary condition if the structure of b is known, but here one continue a duscussion about the properties of the weighting matrix under the sufficient conditions obtained above.

Inserting eqs. (3.5a), (3.5b), (3.8a) and (4.3a) into Riccati equation of eq. (3.10) generates the following four equations :

$$\widetilde{A}'P_1+\widetilde{A}'_\tau P'_2+P_1\widetilde{A}+P_2\widetilde{A}_\tau+0_1=(w_a\widetilde{b}'P_1+(w_a\widetilde{b}'_\tau+w_b\widetilde{b}')P'_2)/\xi \quad \cdots\cdots (4.6)$$

$$\widetilde{A}'P_2+\widetilde{A}'_\tau P_3+P_2\widetilde{A}+0_2=(w_a\widetilde{b}'P_2+(w_a\widetilde{b}'_\tau+w_b\widetilde{b}')P_3)/\xi \quad \cdots\cdots (4.7)$$

$$\widetilde{A}'P'_2+P_3\widetilde{A}'_\tau+P'_2\widetilde{A}+0'_2=(w_c\widetilde{b}'P_1+(w_c\widetilde{b}'_\tau+w_d\widetilde{b}')P'_2)/\xi \quad \cdots\cdots (4.8)$$

$$\widetilde{A}'P_3+P_3\widetilde{A}+0_3=(w_c\widetilde{b}'P_2+(w_c\widetilde{b}'_\tau+w_d\widetilde{b}')P_3)/\xi \quad \cdots\cdots (4.9)$$

where

$$w_a=\beta^2 r(P_1\widetilde{b}+P_2\widetilde{b}_\tau)-\rho rP_2\widetilde{b}$$

$$w_c=\beta^2 r(P'_2\widetilde{b}+P_3\widetilde{b}_\tau)-\rho rP_3\widetilde{b}$$

$$w_b=-\rho r(P_1\widetilde{b}+P_2\widetilde{b}_\tau)+rP_2\widetilde{b}$$

$$w_a = -\rho r(P'_2\tilde{b} + P_3\tilde{b}_\tau) + rP_3\tilde{b}$$

From the eqs. (4.6)~(4.9), the weighting matrices are derived as (Appendix) :

$$Q_{12} = Qn + M_1(An - nE)$$
$$\rho Q_{14} = \rho Qn + L_1(An - nE)$$
$$\beta^2 Q_{34} = \beta^2 Qn + 0_1(An - nE) \qquad \cdots\cdots (4.10)$$

with $L_1 A_\tau n = M_1 A_\tau n = 0_1 A_\tau n = [0]$, i.e.,

$$L_1 A_\tau n = \begin{pmatrix} L_1 & L_2 \\ L'_2 & L_3 \end{pmatrix} \begin{pmatrix} 0 & \begin{matrix} X \\ \vdots \\ X \end{matrix} \end{pmatrix} \begin{pmatrix} I_{n-1} \\ 0 \end{pmatrix} = 0 \qquad \cdots\cdots (4.11)$$

and

$$An - nE = \begin{pmatrix} & \alpha_0 \\ & \alpha_1 \\ 0 & \vdots \\ & \alpha_{n-2} \\ & 1 \end{pmatrix} \qquad \cdots\cdots (4.12)$$

As is known from the eq. (4.10), the weighting matrice Q_{12}, Q_{14}, and Q_{34} are obviously a function of eigenvalues of observer. If the observer eigenvalues become large these weighting matrices also become large, that is, it can be said that the augumented performance index defined in section 3 contains an information as to the observer characteristics.

The rest weighting matrices Q_a, Q_b and Q_c are so selected to satisfy that the weighting matrix $\underset{\sim}{Q}$ is to be positive definite.

With the determined Q_a, Q_b and Q_c the matrix M_3 is directly obtained from the eq. (A−7).

5. Conclusion

This paper is studied with respect to the new and refined approach to the design of optimal regulator when some of state variables are not measured directly.

The optimal control is synthesized in such a way as to minimize not only the basic cost function but also the generalized performance index with the consideration of the trajectory sensitivities.

Furthermore, it is discussed for what performance index the control law using the estimates of the state variables is optimal and for the relation between the weighting matrices and the eigenalues of the observer, and a sufficient condition is derived.

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Appendix

Substituting the partitioned from ofmatrices $\widetilde{A}$, $\widetilde{A}_\tau$, and $\underline{Q}$ into the eq. (4.6), it follows that,

$$\begin{bmatrix} A'M_1 & A'M_2 \\ E'M'_2 & E'M_3 \end{bmatrix}+\begin{bmatrix} A'L'_1 & A'L_2 \\ 0 & 0 \end{bmatrix}+\begin{bmatrix} M_1A & M_2E \\ M'_2A & M_3E \end{bmatrix}+\begin{bmatrix} L_1A & 0 \\ L'_2A & 0 \end{bmatrix}+\begin{bmatrix} Q & Q_{12} \\ Q'_{12} & Q_a \end{bmatrix}$$

$$=\frac{1}{\xi}\begin{bmatrix} k_{a1} & k_{a2} \\ k'_{a2} & k_{a3} \end{bmatrix} \quad \cdots\cdots\cdots(A-1)$$

where

$$k_{a1}=(\overbrace{\beta^2rM_1bb'+\beta^2rL_1b_\tau b'-\rho rL_1bb'}^{\Delta_1})M_1+(\overbrace{\beta^2rM_1bb'_\tau+\beta^2L_1b_\tau b'_\tau-\rho rL_1bb'_\tau}^{\Delta_2})L'_1$$

$$+(\overbrace{rL_1bb'-\rho rM_1b_\tau b'-\rho rL_1b_\tau b'}^{\Delta_3})L'_1$$

$$k_{a2}=(\quad\Delta_1\quad)M_2+(\quad\Delta_2\quad)L_2$$

$$+(\quad\Delta_3\quad)L_2$$

$$k_{a3}=(\beta^2rM'_2bb'+\beta^2rL'_2b_\tau b'-\rho rL'_2bb')M_2+(\beta^2rM'_2bb'_\tau+\beta^2rL'^2b_\tau b'_\tau$$

$$-\rho rL'_2bb'_\tau)L_2+rL'_2bb'-\rho rL'_2b_\tau b')L_2$$

Using the sufficient conditions of eq. (4.5)and eq. (A-1) together, the following result is obtained:

$$Q_{12}=Qn+M_1(An-nE)+L_1A_\tau n \quad \cdots\cdots\cdots(A-2)$$

similarly, eq. (4.7) is,

$$\begin{bmatrix} A'L_1 & A'L_2 \\ E'L'_2 & E'L_3 \end{bmatrix}+\begin{bmatrix} A'_\tau 0'_1 & A'_\tau 0_2 \\ 0 & 0 \end{bmatrix}+\begin{bmatrix} L_1A & L_2E \\ L'_2A & L_3E \end{bmatrix}+\begin{bmatrix} \rho Q & \rho Q_{14} \\ 0'_{14} & Q_b \end{bmatrix}=\frac{1}{\xi}\begin{bmatrix} k_{b1} & k_{b2} \\ k'_{b2} & k_{b3} \end{bmatrix}$$

$$\cdots\cdots\cdots(A-3)$$

where

$$k_{b1}=(\overbrace{\beta^2rM_1bb'+\beta^2rL_1b_\tau b'-\rho rL_1bb'}^{\pi_1})L_1+(\overbrace{\beta^2rM_1bb'_\tau+\beta^2rL_1b_\tau b'_\tau-\rho rL_1bb'_\tau}^{\pi_2})0_1$$

$$+(\overbrace{rL_1bb'-\rho M_1bb'-\rho L_2b_\tau b'}^{\pi_3})0_1$$

$$k_{b2}=(\quad\pi_1\quad)L_2+(\quad\pi_2\quad)0_2$$

$$+(\quad\pi_3\quad)0_2$$

$$k_{b3}=(\beta^2rM'_2bb'+\beta^2rL_1b_\tau b'-\rho rL'_2bb')L_2+(\beta^2rM_2bb'_\tau+\beta^2rL'_2b_\tau b'_\tau-\rho rL'_2bb'_\tau)0_2$$

$$+(-\rho rM'_2bb'-\rho rL'_2b_\tau b'+rL'_2bb')0_2$$

and eq. (4.9) is,

$$\begin{bmatrix} A'0_1 & A'0_2 \\ E'0'_2 & E'0_3 \end{bmatrix}+\begin{bmatrix} 0_1A & 0_2E \\ 0'_2A & 0_3E \end{bmatrix}+\begin{bmatrix} \beta^2Q & \beta^2Q_{34} \\ \beta_2Q'_{34} & \beta_2Q_c \end{bmatrix}=\frac{1}{\xi}\begin{bmatrix} k_{d1} & k_{d2} \\ k_{d2} & k_{d3} \end{bmatrix} \cdots\cdots\cdots(A-4)$$

where

$$k_{d1}=(\overbrace{\beta^2rL'_1bb'+\beta^2r0_1b_\tau b'-\rho r0_1bb'}^{\varepsilon_1})L_1+(\overbrace{\beta^2rL'_1bb'_\tau+\beta^2r0_1b_\tau b'_\tau-\rho r0_1bb'_\tau}^{\varepsilon_2})0_1$$

$$+(\overbrace{-\rho rM'_2bb'-\rho rL'_2b_\tau b'+rL'_2bb'}^{\varepsilon_3})0_2$$

$$k_{d2}=(\qquad \varepsilon_1 \qquad)L_2+(\qquad \varepsilon_2 \qquad)0_2$$
$$+(\qquad \varepsilon_3 \qquad)0_2$$
$$k_{d3}=(\beta^2 rL'_2bb'+\beta^2 r0'_2b_\tau b'-\rho r0'_2bb')L_2+(\beta^2 rL'_2bb'+\beta^2 r0'_2bb'-\rho r0'_2bb')0_2$$
$$+(-\rho rL'_2bb'-\rho r0'_2bb'-r0'_2bb')0_2$$

with the same way, the following results are easily obtained:

$$\rho Q_{14}=\rho Qn+L_1(An-nE) \qquad \text{(A-5)}$$
$$\beta^2 Q_{34}=\beta^2 Qn+0_1(An-nE) \qquad \text{(A-6)}$$

Furthermore, the upper left, the upper right and the lower right part of the equations (A-1), (A-3) and (A-4) together imply,

$$A'_fP_f+P_fA_f+Q_f=P_fb_fR_f^{-1}b'_fP_f \qquad \text{(A-7)}$$
$$A'_fP_g+P_gE+Q_g=P_fb_fR^{-1}b'_fP_g \qquad \text{(A-8)}$$
$$E'P_h+P_hE+Q_h=P_gb_fR^{-1}b'_fP_g \qquad \text{(A-9)}$$