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DYNAMICS OF TENSION LEG PLATFORM TETHERS AT LOW TENSION PART 2 - COMBINED EXCITATION

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Abstract

In conventional design practice, tension leg platform (TLPs) tethers are designed to be highly tensioned. This high pre-tension is, however, a significant restriction to the payload growth of TLPs. This paper reports on the second stage of an investigation into the vibrations of tethers at low tension. Unlike conventional approaches for slender marine structures, tethers are considered to be simultaneously subjected to time-varying axial forces (parametric excitation) and lateral forces (forcing excitation). Therefore, the tethers can be regarded as dynamic systems under combined excitation. In order to solve this combined excitation problem, the governing partial differential equation of lateral motion of a tether is reduced to a nonlinear differential equation. This nonlinear equation of combined excitation is solved by using both Romberg's method and the fourth-order Runge-Kutta method together.

It is found that taking account of combined excitation is of little importance compared to forcing excitation in the first instability region of the Mathieu stability chart. However, from the second instability regions onwards, that is, when the natural periods of tethers are equal or shorter than excitation periods, combined excitation becomes significant. Therefore, since the dynamic operating conditions of most tethers correspond to higher-order instability regions, it is necessary to consider combined excitation in the dynamic analysis of tethers.

The results of theoretical developments on combined excitation are applied to example tethers and used to show that from the third instability regions onwards, tether lateral response amplitudes increase with pre-tension reduction, in the second instability region, the amplitudes do not change, whereas, in the first instability region, the response amplitudes rather decrease with pre-tension reduction.

INTRODUCTION

A tension leg platform (TLP) is one of the most promising and potentially cost effective solutions for hydrocarbon production in deeper waters. Figure 1 shows some existing and planned TLPs. Although the design characteristics of those TLPs are different from each other, their design pre-tension criterion is a common feature. These tethers are conventionally designed to be highly tensioned so as not go slack in a hundred year extreme sea condition. This aspect, however, causes a critical disadvantage of these TLP systems. In other words, in order to keep such high pre-tension, deck payloads of TLP systems are significantly restricted. Actually, due to this deck payload limitation, hydrocarbon field development is being carried out by using other compliant structures such as guyed towers and semisubmersibles (see Reference No. 16). Therefore, the possibility of increasing payload over conventional designs needs to be investigated further by examining the feasibility of reducing pre-tension in both existing and forthcoming TLP designs. There are other reasons also for studying low tension tethers - these are that the failure probability of a tether subjected to maximum tension is higher than that of one with minimum tension (Harding and Banon, 1989) and that conventional environmental design conditions for tether pre-tension are too conservatively considered (Mercier, 1989).

This paper reports the second stage of an investigation into the dynamics of tethers with reduced pre-tension which would involve the tethers being adequately tensioned in normal sea states but being slack for short durations in a hundred year extreme wave trough. In this paper, the dynamic behaviour of low tensioned tethers at such extreme sea conditions is studied. Unlike conventional dynamic analysis of slender marine structures, tethers here are modelled to be simultaneously subjected to time-varying axial forces (called parametric excitation) and lateral forces (called forcing or external excitation). The time-varying axial forces are considered to be imparted by surface platform heave motion and the lateral forces are induced by platform surge motion. These tethers are, therefore, regarded as a system under combined parametric and forcing excitation.

In reality, although most slender marine structures as well as TLP tethers are usually simultaneously subjected to combined forcing and parametric excitation, the two excitations have been separately considered. Most dynamic analysis of slender marine structures have been concerned with forcing excitation - see, for example, Young et al. (1978), Kirk et al. (1979) and Triantafyllou et al. (1983). There have been also other investigations into parametric excitation, i.e., the Mathieu stability problem of slender marine structures by Hsu (1975), Strickland and Mason (1988), and Patel and Park (1991).

Compared to forcing or parametric excitation, research on combined excitation has only been recently carried out in the last two decades. Systems under combined excitation can often arise in engineering or applied science. Examples of these are beams which are simultaneously subjected to transverse support motions and axial forces, mechanisms on vibrating foundations, a pendulum whose support oscillates at an angle with the vertical and a turbine blade connected to a whirling shaft. Most slender marine structures such as marine risers and TLP tethers also fall under these kinds of dynamic systems.

There has been some literature on combined excitation problems from Hsu and Cheng (1974), Troger and Hsu (1979), HaQuang and Mook (1987) and Plaut et al. (1990). However, there has not been previous work which is concerned with structural vibrations including nonlinear hydrodynamic damping or dynamic conditions corresponding to higher-order instability region for tensioned slender marine structures. Therefore, it has been motivated to examine the effect of combined excitation on the dynamic response of a TLP tether which is a typical example of slender marine structures under combined excitation.

THEORETICAL APPROACH

The tether of a TLP is considered as a straight, simply supported column of uniform cross section. Figure 2 shows the tether model under combined excitation and segment notation. For this kind of marine structures, a governing equation has been derived by Young and Fowler (1978) and Chakrabarti and Frampton (1982). The axial tension variation along the tether is taken to be constant, which is true for neuturally buoyant tethers. Then, the governing equation of lateral motion of a tether can be written in the form by adding time-varying axial forces to constant tension,

$$M\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} - (T_0 - S\cos\omega t)\frac{\partial^2 y}{\partial x^2} + B_v\left|\frac{\partial y}{\partial t}\right|\frac{\partial y}{\partial t} = 0 \tag{1}$$

where M is the total mass per unit length of the tether segment, EI is the tether flexural rigidity, T_0 is constant axial tension, S is the time-varying axial force amplitude, ω is the angular frequency of the surface platform heave motion and $B_v = 0.5\, C_D\, \rho_w\, d_o$, where again C_D is a drag coefficient, d_o is the outer diameter of the tether and ρ_w is sea water density. The heave motion of the surface platform which induces time-varying axial forces is assumed to be sinusoidal for analytical simplicity. As can be seen later, the lateral response amplitudes of tethers are limited to be small compared to the tether length by the hydrodynamic damping force. Therefore, geometric nonlinearity is not considered in this work.

In order to obtain the response of lateral motion of the tether, the partial differential equation (1) needs to be solved. The method of separation of variable is used to reduce the partial differential equation to a simple ordinary differential equation. By looking at Figure 2, it can be seen that there exist two normal functions; one is a rigid body mode and the other are sinusoidal elastic response modes. These mode shapes are based on the fact that both ends of the tether are assumed to be pin jointed. Therefore, according to time-dependent boundary condition theory (Mindlin and Goodman 1950), an approximate solution to equation (1) can be assumed in the form

$$y(x,t) = h(t)\frac{x}{L} + \sum_{n=1}^{\infty} f_n(t) \sin \frac{n\pi x}{L} \qquad (2)$$

where, $f_n(t)$ is an unknown function of the elastic response modes, which should be obtained from the following analysis and h(t) is a prescribed lateral movement of the top end of the tether imparted by the surface platform surge motion. Here h(t) needs to be incorporated with the time-varying axial force, $-S \cos \omega t$. The initial boundary condition of the top end is assumed to be in the middle point of surge motion and in the lowest position of heave motion. In addition, the top end is assumed to rotate in the clockwise direction by the wave-induced surface platform motion. Figure 3 shows the profiles of combined forcing (top end lateral displacement) and parametric (time-varying axial forces) excitation for clarification. Therefore h(t) can be assumed to be

$$h(t) = - y_o \sin \omega t \qquad (3)$$

where y_o and ω are the lateral displacement amplitude and the angular frequency of the top end motion. The reason for the assumption of sinusoidal motion is the same as that for the previous time-varying axial forces.

Substituting equations (2) and (3) into equation (1) gives

$$\sum_{n=1}^{\infty} \left\{ \left[EI \left(\frac{n\pi}{L}\right)^4 + (T_0 - S \cos \omega t)\left(\frac{n\pi}{L}\right)^2 \right] \sin \frac{n\pi}{L} \right\} f_n$$
$$+ M \sum_{n=1}^{\infty} \ddot{f}_n \sin \frac{n\pi x}{L} + B_v \left| -\frac{x}{L} y_0 \omega \cos \omega t + \sum_{n=1}^{\infty} \sin \frac{n\pi x}{L} \frac{df_n}{dt} \right| \cdot$$
$$\left\{ -\frac{x}{L} y_0 \omega \cos \omega t + \sum_{n=1}^{\infty} \sin \frac{n\pi x}{L} \frac{df_n}{dt} \right\} = -\frac{xM}{L} y_0 \omega^2 \sin \omega t \qquad (4)$$

Following Galerkin's variational method, Equation (3) is multiplied throughout by $\sin m\pi x/L$ and integrated over the length of the model, then

$$\frac{ML}{2}\frac{d^2f_m}{dt^2}+\left[EI\left(\frac{m\pi}{L}\right)^4\frac{L}{2}+(T_0-S\cos\omega t)\left(\frac{m\pi}{L}\right)^2\frac{L}{2}\right]f_m$$
$$+B_v\int_0^L\left|-\frac{x}{L}y_0\,\omega\cos\omega t+\sin\frac{m\pi x}{L}\frac{df_m}{dt}\right|\cdot\left\{-\frac{x}{L}y_0\,\omega\cos\omega t+\sin\frac{m\pi x}{L}\frac{df_m}{dt}\right\}\cdot$$
$$\sin\frac{m\pi x}{L}dx = \frac{LM}{m\pi}(-1)^m\,y_0\,\omega^2\sin\omega t \tag{5}$$

In deriving the above equation, a mode coupling effect in the quadratic nonlinear damping term is neglected and the following integrations are used.

$$\int_0^L x\sin\frac{m\pi x}{L}dx = -\frac{L^2}{m\pi}(-1)^m$$
$$\int_0^L \sin\frac{n\pi x}{L}\sin\frac{m\pi x}{L}dx = \frac{L}{2} \quad \text{for} \quad n=m$$
$$= 0 \quad \text{for} \quad n\neq m \tag{6}$$

Rearranging equation (5) gives

$$\frac{d^2f_m}{dt^2}+(\varpi_m^2-c_1\cos\omega t)f_m+\frac{2B_v}{ML}\int_0^L\left|-\frac{\omega y_0 x}{L}\cos\omega t+\sin\frac{m\pi x}{L}\frac{df_m}{dt}\right|\cdot$$
$$\{-\frac{\omega y_0 x}{L}\cos\omega t+\sin\frac{m\pi x}{L}\frac{df_m}{dt}\}\sin\frac{m\pi x}{L}dx=(-1)^m\frac{2y_0\omega^2}{m\pi}\sin\omega t \tag{7}$$

where

$$\varpi_m^2=\frac{EI}{M}\left(\frac{m\pi}{L}\right)^4+\frac{T_0}{M}\left(\frac{m\pi}{L}\right)^2 \qquad c_1=\frac{S}{M}\left(\frac{m\pi}{L}\right)^2 \tag{8}$$

ϖ_m in equation (8) indicates the natural angular frequency of m-th mode of the tether. The equation (7) represents vibrations of tethers subjected to combined (forcing and parametric) excitation.

It is also useful to rewrite equation (7) in a nondimensional form with respect to time by using $t=(2/\omega)\tau$. Then,

$$\frac{d^2f_m}{d\tau^2}+(\delta-2q\cos2\tau)f_m+c\int_0^L\left|-\frac{\sqrt{3\pi}}{L\sqrt{L}}y_0x\cos2\tau+\frac{\sqrt{3\pi}}{2\sqrt{L}}\sin\frac{m\pi x}{L}\frac{df_m}{d\tau}\right|\cdot$$
$$\left\{-\frac{\sqrt{3\pi}}{L\sqrt{L}}y_0x\cos2\tau+\frac{\sqrt{3\pi}}{2\sqrt{L}}\sin\frac{m\pi x}{L}\frac{df_m}{d\tau}\right\}\sin\frac{m\pi x}{L}dx=(-1)^m\frac{8y_0}{m\pi}\sin2\tau \tag{9}$$

where δ and q are parameters of the Mathieu equation and c is a hydrodynamic damping related coefficient, and they are given in the form,

$$\delta = \frac{(2\varpi_m)^2}{\omega^2} \qquad q = \frac{2S}{EI(m\pi/L)^2 + T_0}\frac{\varpi_m^2}{\omega^2} \qquad c = \frac{8B_v}{3\pi M} \tag{10}$$

If f_m is obtained by solving equation (9), the lateral responses of the tethers can be obtained by substituting f_m into equation (2). Unfortunately it is impossible to obtain an exact full analytical solution of equation (9). Therefore, it is necessary to employ a numerical method.

Before carrying out the numerical analysis, it is worthwhile to examine the structure of final equation (9). First, if the time-varying axial force, $S \cos \omega t$, is not considered, the resulting motion of the tethers becomes forced vibrations. In other words, if S is zero, q is zero from equation (10), so equation (9) becomes

$$\frac{d^2 f_m}{d\tau^2} + \delta f_m + c\int_0^L \left| -\frac{\sqrt{3\pi}}{L\sqrt{L}} y_0 x \cos 2\tau + \frac{\sqrt{3\pi}}{2\sqrt{L}} \sin \frac{m\pi x}{L}\frac{df_m}{d\tau}\right| \cdot \left\{ -\frac{\sqrt{3\pi}}{L\sqrt{L}} y_0 x \cos 2\tau + \frac{\sqrt{3\pi}}{2\sqrt{L}} \sin \frac{m\pi x}{L}\frac{df_m}{d\tau}\right\} \sin \frac{m\pi x}{L} dx = (-1)^m \frac{8y_0}{m\pi} \sin 2\tau \tag{11}$$

An analytical solution of equation (11) for forced vibrations can be obtained by iteration. However, if the hydrodynamic damping effect is excluded, that is, $c = 0$, the solution becomes of the form

$$f_m(\tau) = \frac{(-1)^m 8y_0}{(\delta - 4)m\pi} \sin 2\tau \qquad \text{or} \qquad f_m(t) = \frac{(-1)^m 2y_0\omega^2}{(\omega_m^2 - \omega^2)\, m\pi} \sin \omega t \tag{12}$$

This result corresponds to undamped forced vibrations of tethers. As can be seen from equation(12), the amplitude of $f_m(t)$ depends on the top end displacement amplitude, y_0, excitation and natural angular frequency, ω and ϖ_m, and mode number, m. A resonance occurs when the forcing angular frequency, ω, comes close to any of the natural frequencies, ϖ_m However when the hydrodynamic damping force is considered, even if the external frequency is equal to any of natural frequencies, the amplitudes of resonance responses is limited as will be seen later.

Secondly, if the lateral motion of the top end is neglected, in other words, if y_0 is zero, equation (9) becomes

$$\frac{d^2 f_m}{d\tau^2} + (\delta - 2q\cos 2\tau)\, f_m + c\left|\frac{df_m}{d\tau}\right|\frac{df_m}{d\tau} = 0 \tag{13}$$

Equation (13) is the nonlinear Mathieu equation and describes parametric vibrations of tethers. An approximate analytical solution can be obtained for small values of the parameters, δ and q, by using perturbation techniques but a numerical method is necessary for larger values of the parameters. The response characteristics of the parametric excitation are known to be as follows ;

(1) When hydrodynamic damping force is excluded, the solutions become stable or unstable according to the combination of parameters, δ and q as can be seen from Figure 4.

(2) When hydrodynamic damping force is included, even unstable solutions are limited.

(3) The response amplitudes of parametric vibrations rely on the natural and excitation frequencies, ϖ_m and ω, the amplitude of parametric excitation, q and hydrodynamic damping, c.

(4) The response frequency (ω_r) of the equation (13) depends instability regions and can be set such as $\omega_r = (N/2)\,\omega$, where N indicates the ordinal of instability regions. Thus the response frequencies in the higher instability region become higher.

Meanwhile, combined excitation problem of equation (9) comprises the characteristics of the above forcing and parametric excitations together. Bearing in mind the characteristics of each vibration, the response pattern of structures under combined excitation are to be obtained by a numerical method for given values of δ , q, c, L and y_0.

NUMERICAL SOLUTION FOR EXAMPLE TETHERS

Equation (9) is solved using the fourth-order Runge-Kutta method with an extension to take account of the integral term in equation (9). Such an extension can use either the trapezoidal rule, Simpson's rule, the Romberg method or Gaussian quadrature. It is known that Romberg's method is in many ways better than other methods since it is accurate, simple and computer-oriented. Therefore, in this work, the Romberg method is used with the Runge-Kutta method for solution of equation (9).

Numerical text books such as Cuo (1972) give more details on the Runge-Kutta method and the Romberg method. A computer program is developed based on the fourth-order Runge-Kutta together with a subroutine program which is based on the Romberg method for evaluating the integral term at each time step. Since initial conditions do not influence the steady state solutions of equation (9), the conditions of $f_1(0)=0.1$ and $df_1(0)/d\tau=0.0$

are used. These initial values are taken to coincide with the combined excitation condition shown in Figure 3.

For a case study to illustrate the results of this research, data for example tethers are given in Table 1. These are typical of Snorre TLP tethers in the case of 300m length. Three tether lengths of 300, 760 and 1500 m are chosen and for convenience other dimensions such as pre-tension , tether diameter and so on are taken to be identical. In the following analysis, 300, 760 and 1500 m of tether length will be called CASE I , II and III tethers respectively. The dominant dynamic conditions of CASE I, II and III correspond to the fifth, second and first instability regions of the Mathieu chart respectively as can be seen from Table 2 and Figure 4. Table 2 is obtained from equations (8) and (10) for the first vibration mode of each tether in Table 1.

Since this paper is concerned with the dynamics of TLP tethers at low tension, it is natural to consider the tethers at the worst sea state. A conventional design tension condition is assigned such that at such a worst sea state, the tethers are about to become slack, which corresponds to the amplitude of time-varying axial force being identical to tether pre-tension. This work is, however, focused on the dynamic behaviour of tethers at the tension reduced below conventional tension by increasing deck payload by the same amount as the tension reduction. Here, the reduction percentage of tether tension are chosen to be 5, 10 and 15 %. The amplitude of top end lateral displacement, y_0, is assumed to be 3.0 m which corresponds to RAO (the ratio amplitude operator) being 0.2 for 15.0 m of wave amplitude. In addition, since high mode responses are also important in the case of long tethers, the elastic response modes of tethers are considered up to their fourth mode in the calculation of f_m.

RESULTS AND DISCUSSION

The analysis of combined (forcing and parametric) excitation presented in this paper is more representative of the reality than conventional dynamic analysis of such structures, where forcing and parametric excitation have been separately considered. Therefore, comparisons between forcing, parametric and combined excitation are first made for the dynamic responses of tethers. The comparison is carried out for all CASE I, II and III tethers. The tether tension condition is chosen to be the conventional design tension.

Figure 5(a) shows a comparison between lateral deflections at the mid-point of the tether subjected to forcing, parametric and combined excitations for CASE I (300 m length) tether. The dynamic condition of CASE I corresponds to around the fifth instability regions of the Mathieu stability chart as can be seen from Table 2. These results indicate that the response from combined excitation is much larger than that from forcing or parametric excitation. An interesting point is that even though the responses of forcing or parametric excitation are very small, those of combined excitation are relatively large. This aspect means that the interaction between the two excitations is significant in increasing the total responses of tether lateral displacements. The response period of forcing excitation is still the same as the excitation period. However, the response frequency of parametric is high and this aspect is quite natural as mentioned earlier. In the case of combined excitation, the response frequency is also high due to the included parametric excitation effect.

Figure 5(b) illustrates the results for the CASE II (760 m length) tether whose dynamic condition fall under the second instability region. Responses of combined excitation are also much larger than those of forcing or parametric excitation. The response periods of three excitations are all the same as that of the 15 second excitation period. However, for CASE III (Figure 5(c)) which corresponds to the first instability region, the response pattern is quite different from CASE I and II, that is, the responses of combined excitation are nearly identical to those of forcing excitation. Meanwhile, the response period of forcing and parametric excitations are respectively double and identical to the 15 second excitation period. This result displays quite natural characteristics of forcing and parametric excitations. In the case of combined excitation, its response period is dependent upon the relative strength between forcing and parametric excitations. In Figure 5(a), the response period of the combined excitation is close to that of the forcing excitation. However, according to a separate calculation, when the strength of the forcing excitation, y0 is reduced to 1.0 m, the period of combined excitation becomes double and closes to that of the parametric excitation as will be seen later.

If the effect of combined excitation is not significant in the other regions as well as in first instability region, the analysis of simple forced vibrations only is satisfactory without dealing with the complex combined excitation problem. However, as already mentioned, the dominances of the three excitations are quite different each other according to instability regions in which they occur. Therefore, more realistic combined excitation should be considered in the dynamic analysis of tethers.

Figure 6 shows more clearly the relative importance of forcing , parametric and combined excitations according to instability regions, i.e., with the values of δ varied. In this research, a hydrodynamic damping force is considered, so the responses are all limited, otherwise they are not limited at unstable conditions. Bearing in mind that $\delta = (2\varpi_m/\omega)^2$ in equation (10), the response pattern of forcing excitation is easily found in textbooks of vibration theory. However those of parametric or combined excitation has been scarcely seen in open literatures. In the case of parametric excitation, as can be seen from Figure 6(a) and Figure 4, relatively large responses occur in each instability region. The largest magnitude exist in the centre of each instability region. By comparing Figure 6 (a) and (b), it is seen that the response of parametric excitation is more dependent upon high vibration mode effects than other excitations.

Beyond the first instability , the responses of combined excitation are much larger than those of forcing excitation, especially in the second and fourth instability regions. In addition, the response amplitudes of combined excitation are much larger than those of parametric excitation. This result means the reduction of the lateral excursion of the top end by adopting a supplemental lateral mooring system (see Figure 1(e)), can significantly reduce the lateral response of the tethers. The reason is that if the top end lateral excursion is reduced which means the strength of forcing excitation is diminished, the dynamic system of tether lateral motion is more close to parametric vibrations. As can be seen from Figures 5 and 6, the effect of parametric excitation on the increase of tether lateral displacement is smaller than that of forcing excitation in so far as S/T0 is equal or less than 1.0.

The results of Figures 5 and 6 clearly demonstrate that consideration of combined excitation is necessary in the dynamic analysis of tethers. Therefore, in the following investigation into the dynamics of low tension tethers, only the combined excitation is considered. Figure 7 illustrates time histories of lateral displacements at the mid-point of example tethers subjected to combined excitation at several tension conditions. Figure 7(a) corresponds to the results of CASE I (300 m length) tether. At a conventional design tension, that is, when time-varying axial force amplitude is equal to the pre-tension, the maximum displacement is about 3 m. When 5, 10 and 15 % of the conventional tension are reduced, the lateral displacements gradually increase to about 4, 5.3, and 6.7m respectively.

Figures 7(b) and 7(c) show the time histories of the lateral displacements of the CASE II (760 m length) and III (1500 m length) tethers respectively. By looking at Figure 7(b) for the CASE II tether, it is found that the pre-tension reduction does not affect the response amplitudes of tether lateral motion, that is, the total displacement of lateral deflections at conventional design tension is about 8 m and even at reduced pre-tension, the magnitude is nearly identical to this value. The result for the CASE III tether (Figure 7(c)) shows a quite different response pattern from CASE I and II, that is, the response amplitudes of lateral displacements decrease as the pre-tension reduces. This aspect is very interesting for the designers of deep water TLP systems. Another interesting aspect is that the response amplitude of CASE III TLP tethers is smaller than CASE II TLP tethers. The largest magnitude of lateral displacement in steady state is about 5.5 m and happens at the conventional design tension.

One of the purposes of this work is to examine the feasibility of operating a TLP system at low tension to overcome the practical disadvantage of relatively small deck pay load capability. Therefore, lateral displacements of tethers are obtained for several reduced tension conditions. If the lateral displacements of tethers are small at the reduced tension, low tension tethers can be probably adopted. The reason is that due to the small response amplitudes, the bending stress of tethers is small and coupling between tethers and a surface platform is not significant. Moreover, even though the pre-tension of the tethers is reduced, the heave motion of the surface platform will not change because the pre-tension magnitude does not affect the natural frequency of the surface platform heave motion. The tether pre-tension reduction causes only the natural frequencies of surge motion of the surface platform to be lower, which is then further away from the zone of high energy of ocean waves.

Therefore, for the above CASE I and III tethers, a certain amount of pre-tension can be reduced over conventional design tension. However, for CASE II, the lateral displacements of tethers are somewhat large even at conventional design tension, so careful consideration is necessary. If pre-tension is reduced, the coupling between the surface platform and tether motion becomes significant and the problem of tether collision will arise. It is not easy to suggest a definite low tension criterion by considering only combined excitation for CASE II. Therefore in addition to combined excitation analysis, further research into coupling between the surface platform and tether motions is necessary for the type of CASE II tethers, i.e., dynamic conditions of tethers corresponding to the second instability region.

The above results are based on the fact that the profile of combined excitation is an exact sinusoid. If the combined excitation profile is random with a narrow banded spectrum, the response amplitudes of lateral deflections are likely to be even smaller. In other words, the above results give more a conservative tension criteria. It is often argued that if tethers go slack, the phenomenon of snap or snatch loading of the tethers occurs, which is significant when the relative velocity of two ends is large. However, in the case of tethers, the anticipated tension loss is not large enough and the surface platform does not acquire the large velocity required for significant snatch loading. Hydrodynamic drag also plays an important role in reducing the adverse effects of snatch loading.

Although this work has clarified the importance of combined excitation in the dynamic analysis of TLP tethers, there remain further works to be studied such as : The effect of other phase lags between forcing and parametric excitation (here, $\pi/2$ is assigned, because the lag between heave and surge motions of surface structure is usually close to $\pi/2$), the interaction between low and high modes of structural vibrations, the effect of different values of hydrodynamic damping and so on.

CONCLUSION

This work has been carried out to investigate the dynamic behaviour of tension leg platform tethers at low tension. The purpose of this study is to find any possibility of increasing deck pay load over conventional design tension conditions. In the theoretical developments, tethers are considered to be simultaneously subjected to forcing excitation (top end lateral movement) and parametric excitation (time-varying axial forces) for simulating more realistic situation. Interesting features are observed comparing results from combined excitation with those from forcing or parametric excitation. In the first region of the Mathieu stability chart, the combined excitation is of little importance compared to forcing excitation. However, from the second instability region onwards, the effect of combined excitation become significant compared to forcing or parametric excitation. Therefore, in the higher-order instability regions, even though one of the forcing or parametric excitation effects is very small, the interaction of two excitations is significant and the response of combined excitation is very large. Since the dynamic conditions of most TLP tethers correspond to higher-order instability regions, consideration of combined excitation is essential for their dynamic analysis.

Applications of the combined excitation problem to some example tethers at low tension provide some valuable results as follows ; (1) In the higher than second instability region, the lateral response amplitudes gradually increase with pre-tension reduction, (2) In the second instability region, the amplitudes do not change, (3) In the first instability region, the response amplitudes decrease with pre-tension reduction. In conclusion, there is a possibility of conventional high tension of TLP tethers being able to be reduced to a certain amount by replacing deck payload increase except for tethers, of which dynamic conditions fall under the second instability region.

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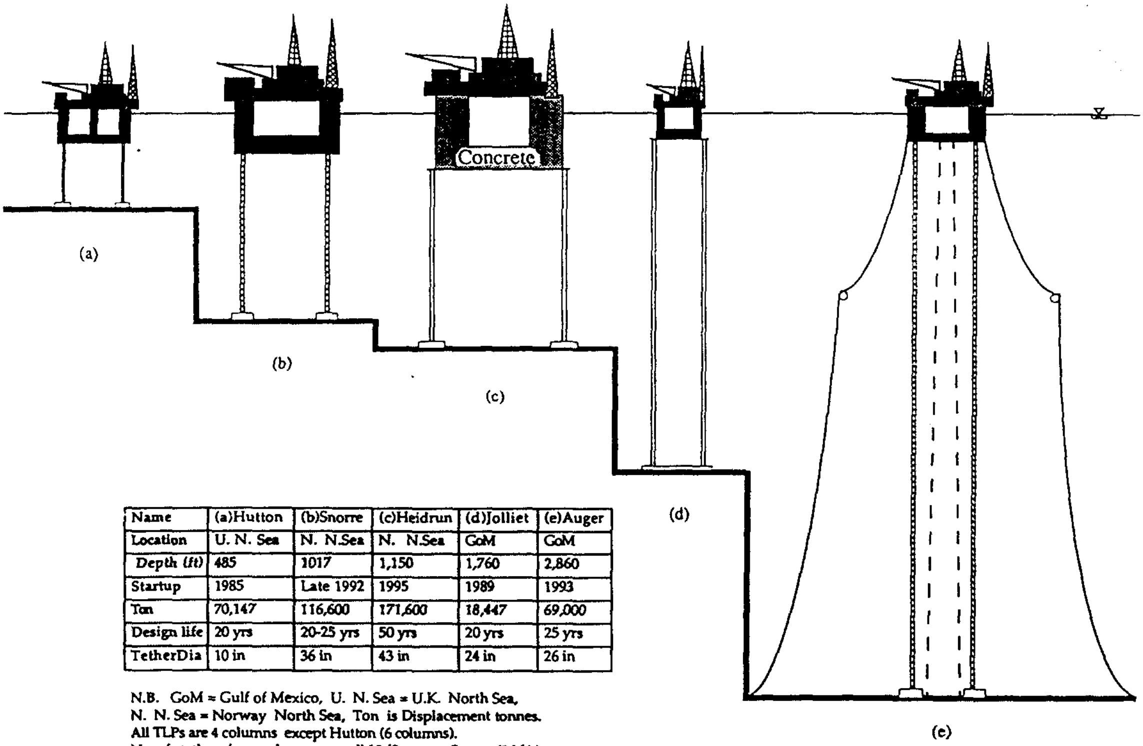
Table 1. Data specification of example TLP tethers for case study

Description	CASE I	CASE II	CASE III
Length, L	300 m	760 m	1500 m
Top tension, T_0	13.0×10^6 N	All the same as CASE I	All the same as CASE I
Flexural rigidity, EI	14.57×10^8 (N m^2)		
Inner diameter, d_i	0.762 m		
Outer diameter, d_o	0.812 m		
Dry mass	726.3 (kg/m^3)		
Damping coefficient, C_d	1.1		
Excitation period ($2\pi/\omega$)	15 seconds		
Time-varying axial force amplitude, S	13.0×10^6 N		
Added-mass coefficient	1.0		
Amplitude of top end displacement, y_o	3.0 m		

Table 2. Values of parameters, δ and q for CASE I, II and III tethers

	CASE I (300 m)	CASE II (760 m)	CASE III (1500 m)
δ	26.11	4.03	1.03
q	12.90	2.01	0.52



Name	(a)Hutton	(b)Snorre	(c)Heidrun	(d)Jolliet	(e)Auger
Location	U. N. Sea	N. N.Sea	N. N.Sea	GoM	GoM
Depth (ft)	485	1017	1,150	1,760	2,860
Startup	1985	Late 1992	1995	1989	1993
Ton	70,147	116,600	171,600	18,447	69,000
Design life	20 yrs	20-25 yrs	50 yrs	20 yrs	25 yrs
TetherDia	10 in	36 in	43 in	24 in	26 in

N.B. GoM = Gulf of Mexico, U. N. Sea = U.K. North Sea,
N. N. Sea = Norway North Sea, Ton is Displacement tonnes.
All TLPs are 4 columns except Hutton (6 columns).
No. of tethers/per column are all 12/3 except Snorre (16/4).

Figure 1. Comparison of existing and planned TLPs.

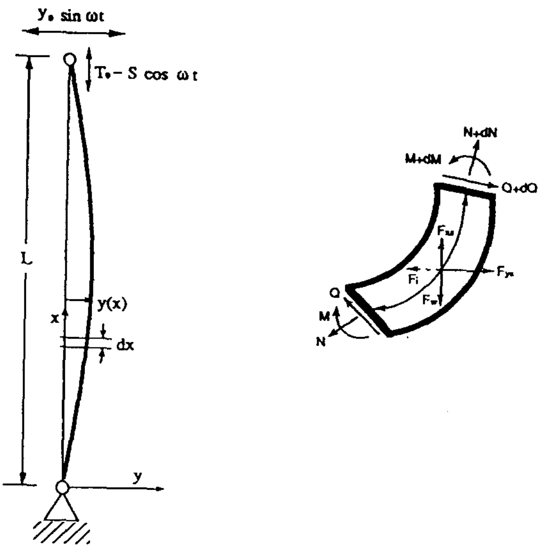


Figure 2. Tether model configuration (a) and Segment notation (b).

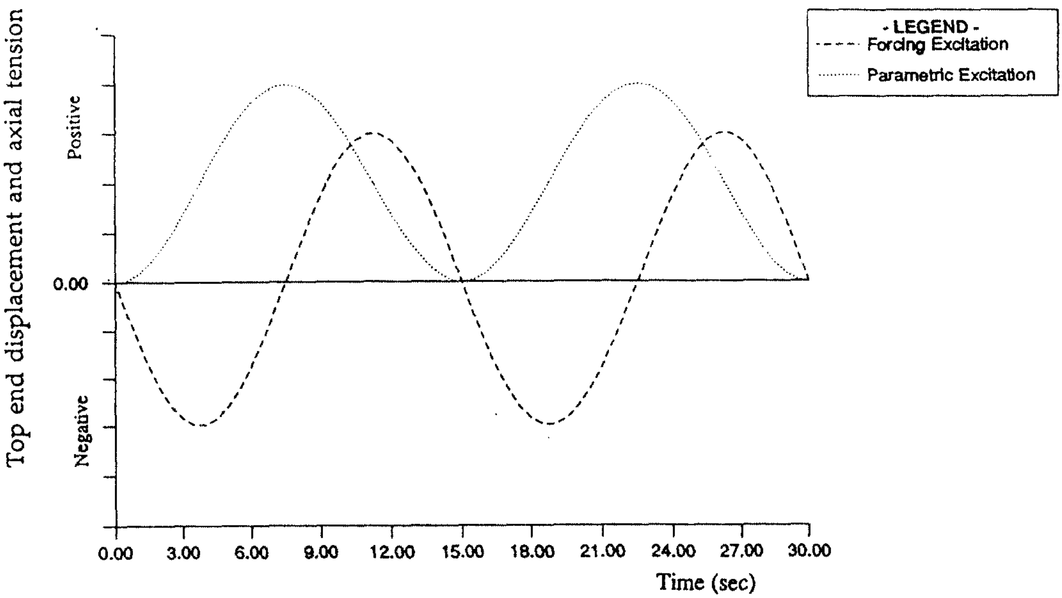


Figure 3. Input profile of combined forcing (- $y_0 \sin \omega t$) and parametric (T_0 - S cos ωt) excitation.

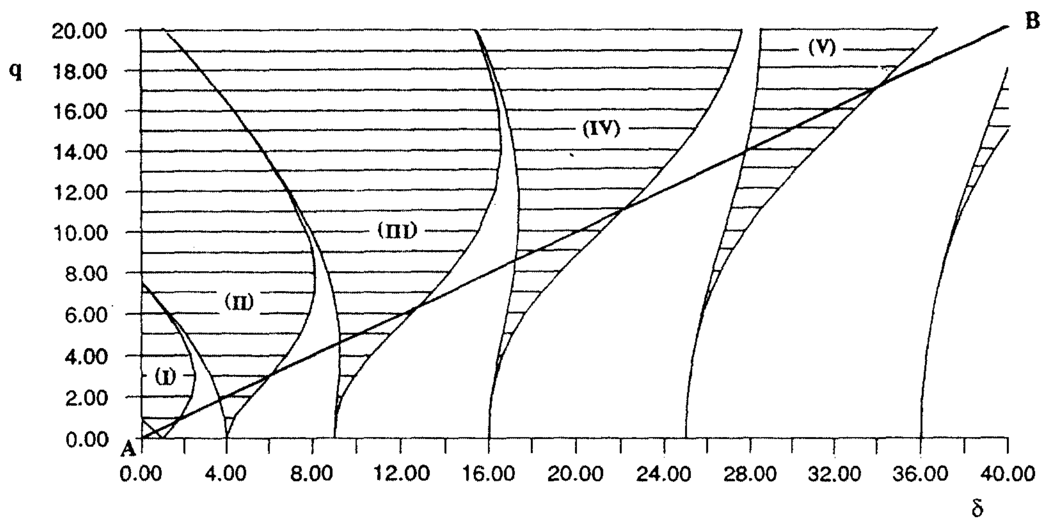
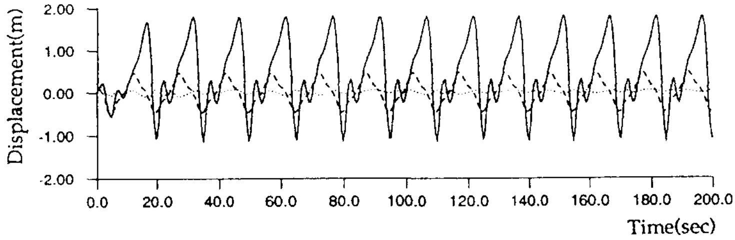
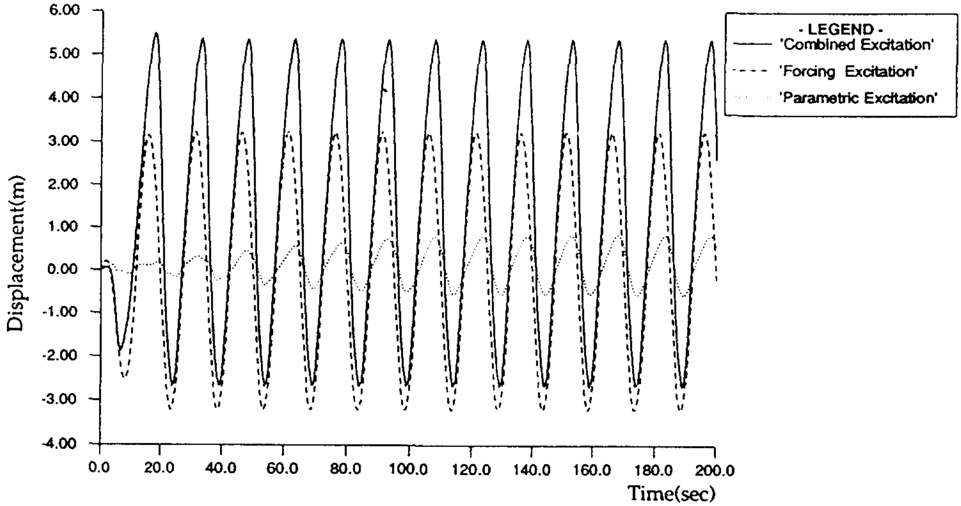


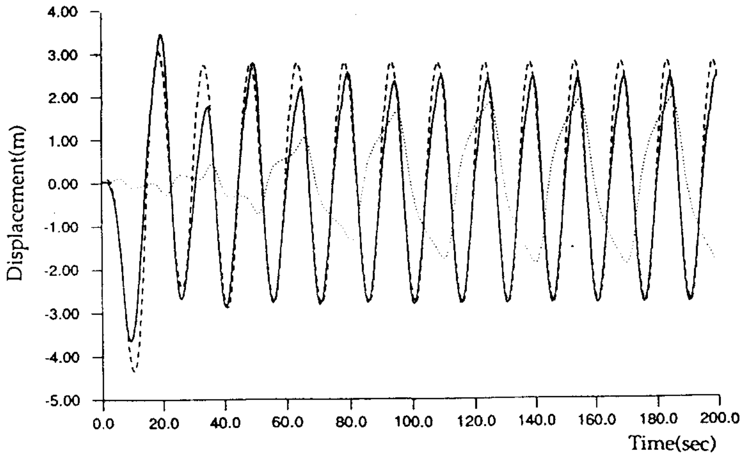
Figure 4. Mathieu stability chart (Shaded areas are unstable)
A B line : Conventional design tension condition ($T_0 = S$)



(a) Fifth instability region (L=300m : δ = 26.11, q = 12.90)

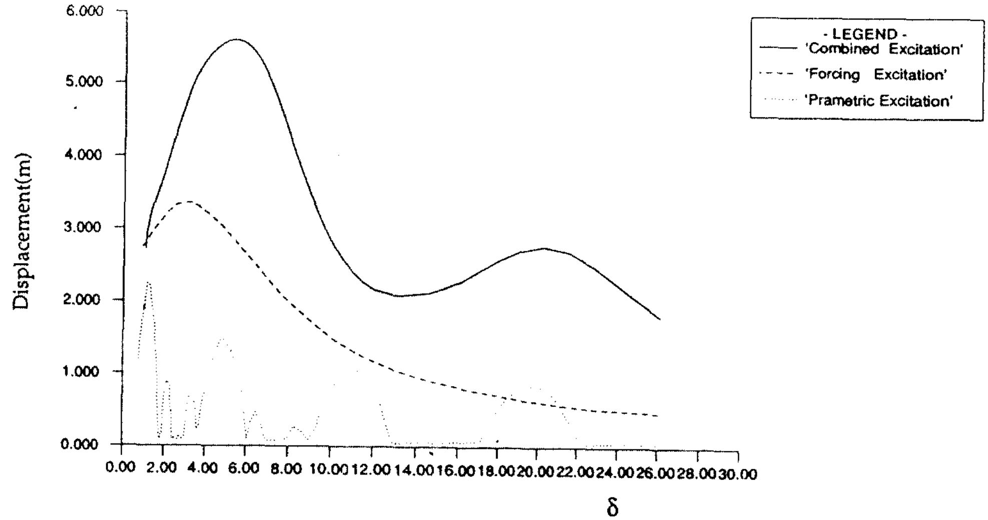


(b) Second instability region (L=760m : δ = 4.03, q = 2.01)

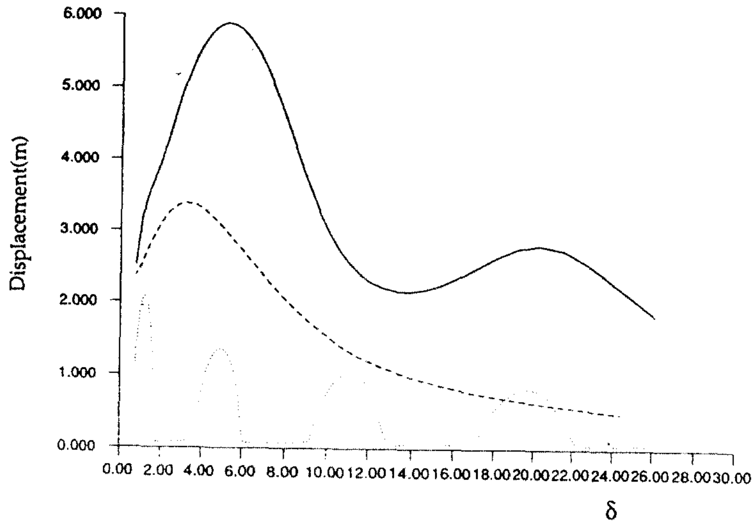


(c) First instability region (L=1500m : δ = 1.03, q = 0.52)

Figure 5. Comparison of lateral displacements of tethers between combined, forcing and parametric excitations.

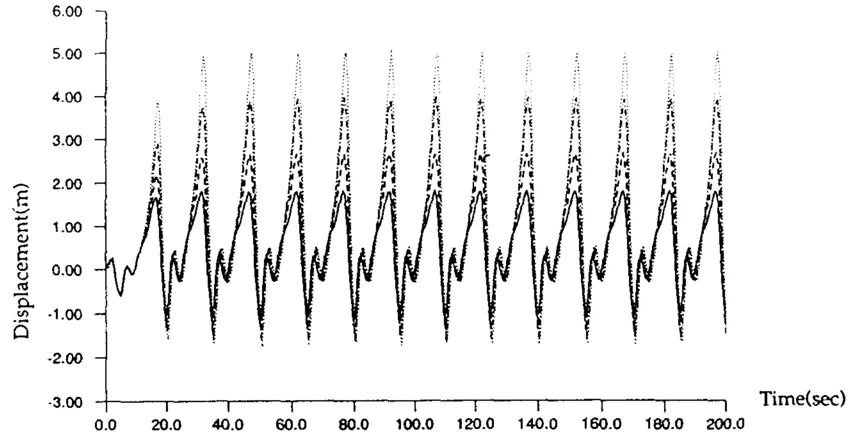


(a) First four modes are considered

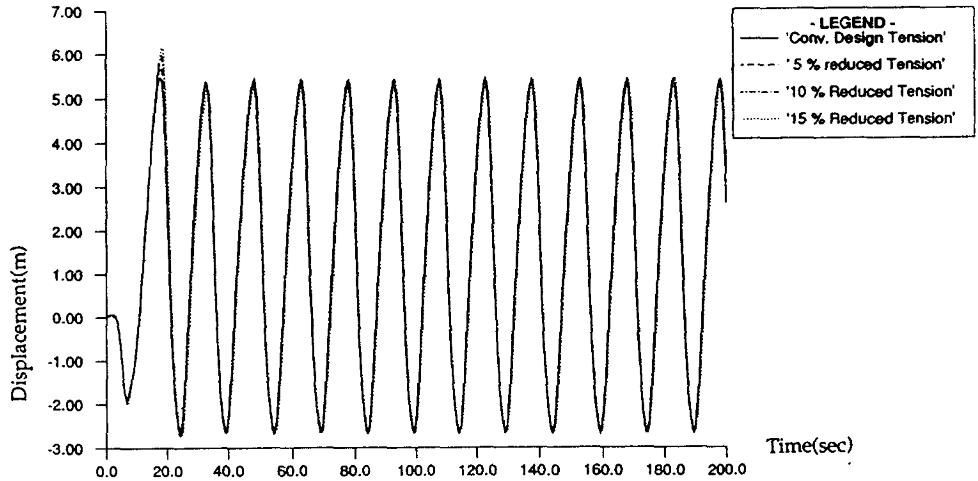


(b) Only first mode is considered

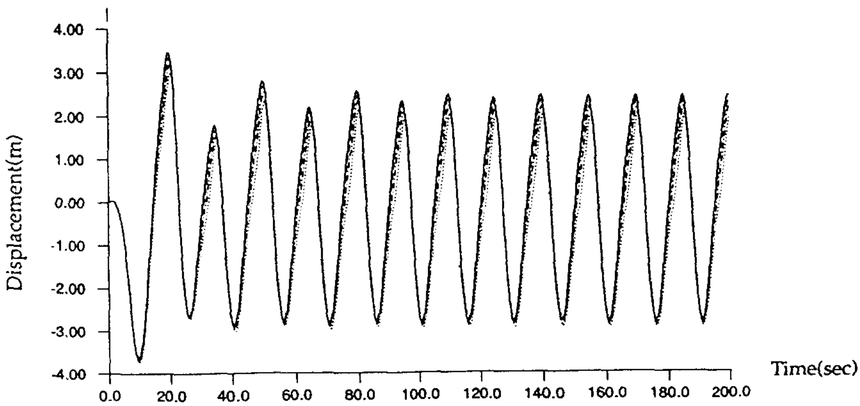
Figure 6. Comparison of lateral displacements of tethers between combined, forcing and parametric excitations with δ variation.



(a) Around Fifth instability region (300 m tether length)



(b) Second instability region (L=760 m tether length)



(c) First instability region (L=1500 m tether length)

Figure 7. Comparison of lateral displacements of tethers subjected to combined excitation for conventionally designed pretension and reduced pretension cases.