Journal of the Research Institute of Basic Sciences,
Korea Maritime University, Vol. 2, 1992.

A Study on Bayes Estimators of Component Steady-state Avaivility

Choon-Il Park*
Jeong-Gi Park**

*Dept. of Applied Mathematics, Korea Maritime Univ., Pusan, Korea
**Dept. of Electronics Engineering, Han Yang Univ., Seoul, Korea

1. Introduction

Let us denote the distrkbution of failure time X and repair time Y by H(X) and G(Y) respectively. Then the repairable component with respect to these to distribution can be determined by steady-state availablity $A=E(X)/(E(X)+E(Y))$, which is the probaility that the repairable component is in operation in the long fraction of time.

Gave and Mazumder[3] investigated the estimation of parameters in case that two probability distributions are specified on the two-state process in operation and under repair. By investigating the failure of previous try and the period of repairs, Nelson[4] determined the interval of prediction for the availability of repairable component. Brender[1] worked on the Bayesian estimation of the steady-state availability by using failure rate and repair rate.

On the other hand, Thompson and Palicio[5] obtained a method to compute the Bayesian interval of the availability of a series or parallel system consisting of several statistically independent two-state subsystems having exponential of failure times and repair times.

The main purpose of this paper is to compute Bayesian estimation of component steady-state availability. In section 2, we will investigate Bayesian estimation of the steady-state availability for noninformative prior density function and in section3, we will compute Bayesian estimation for conjugate prior density function. Finally, some examples to compare several estimations numerically will be given in section 4.

2. Bayes Eatimation for Noninformative Prior Density Function

Consider a repariable component for which the failure time X is distributed as $H(\theta)$ with exponential probability density function(pdf)

$$h(x|\theta) = \frac{1}{\theta}, \exp\left(-\frac{X}{\theta}\right), \quad x>0, \ \theta>0$$

and the repairable time Y is distributed as $G(\alpha,\beta)$ with Gamma-pdf

$$g(y|\alpha_0, \beta_0) = \frac{1}{\Gamma(\alpha_0)\ \beta_0^{\alpha_0}} \cdot y^{(\alpha_0-1)} \cdot \exp\left(-\frac{Y}{\beta_0}\right), \ x>0, \ \alpha>0, \ \beta>0$$

, where α(shape parameter)has a known value α_0, and β(scale parameter) is unknown value β_0.

Assume that X. Y independent the stady·state availability is given by

$$A = \frac{E(X)}{E(X) + E(Y)}, \ E(X) = 0, \ E(Y) = \alpha_0{}^0\beta_0$$

Then, the likelihood function of q, Tz, $T_y$given

$$(2\text{-}1)\ L(q,T_x,T_y|\theta,\beta_0,\alpha_0) = \frac{1}{\theta^q}\exp\left(\frac{T_x}{\theta}\right)\left[\frac{1}{\Gamma(\alpha_0)}\right]^q\left(\frac{1}{\beta_0}\right)^{q\alpha_0}\left(\prod_{i=0}^{q} y_i\right)^{(\alpha_0-1)}\exp\left(-\frac{T_y}{\beta_0}\right),$$

, where q:observed failure repair cycles

$T_x = \sum_{i=1}^{q} X_i$ (X_i is the i-th failure time) : total observed operating time

$T_y = \sum_{i=1}^{q} Y_i$ (Y_i is the i-th failure time) : total observed repair time

Let's introduce some integration for calculation of expectation,

$$(2\text{-}2)\ \int_0^\infty z^{(p-1)} \cdot \exp(-az)dz = \Gamma(p)a^{-p},$$

$$\int_0 z^{(b-1)}(1-z)^{(c-b-1)}(1-tz)^{-a}dz = \frac{\Gamma(b)\ \Gamma(c-b)}{\Gamma(c)}\,{}_2F_1(a,b;\ c,t),$$

for $|t| < 1$, $c > b > 0$ and

$${}_2F_1(a,b;\ c,t) = \sum_{i}^{q} \frac{(a_i)\ (b_i)}{(C_i)\cdot i!}\, t^i$$

is confluent hypergiometric functions of Garss form for $|t| < 1$ with

$$(a_i) = \frac{\Gamma(a+i)}{\Gamma(a)}$$

Form Erdilyi[2], we have

$$(2\text{-}3)\quad {}_2F_1(a,b;\ c,z) = (1-z)^{(c-a-b)}\ {}_2F_1(c-a,\ c-b\ ;\ c-z),\ \text{for } |z| < 1\ ;$$

,where(a,b,c -a, c-b) is positive integer.

Assume that MTBF in the exponential failure time distributed and scale parameter in the Gamma repair time distribution has independent noninformative prior distribution given by

$$(2-4)\qquad f(\theta) \propto \frac{1}{\theta^{m1}}, \qquad m_1 > 0$$

$$(2-5)\qquad f(\beta) \propto \frac{1}{\beta_0^{m2}}, \qquad \beta_0 > 0,\ m_2 > 0$$

Then we have the following lemma.

LEMMA 2.1 The joint posterior density function of θ and β is given by

$$f(\theta,\beta_0|q,\ T_x,\ T_y\ ;\ \alpha_0) = \frac{(T_x)^{(q+M1-1)}(T_y)^{(q\alpha_0+m2-1)}\exp[-(T_x/\theta)+(T_x/\theta)]}{\theta^{(q+m1)}\ \beta_0^{(q\alpha_0+m2)}\Gamma(q+m_1-1)\Gamma(q\alpha_0+m_2-1)}$$

, where $\theta > 0$, $\beta > 0$ and $\Gamma(\cdot)$ is a Gamma function.

PROOF. From(2-1), (2-2) and (2-3), the joint posterior distribution of θ, β_0 reduces to

$$(2\text{-}6)\qquad f(\theta,\ \beta_0|q,\ T_x,\ T_y\ ;\ \alpha_0)$$

$$= \frac{L(q,\ T_x,\ T_y|\theta,\ \beta_0:\alpha_0)\cdot f_1(\theta)\ f_2(\beta_0)}{\int_0^X \int_0^X L(q,\ T_x,\ T_y|\theta,\ \beta_0:\alpha_0)\cdot f_1(\theta)\ f_2(\beta_0)d\beta_0 d\theta}$$

$$\frac{e^{-q}\exp(-\frac{T_x}{\theta})\left[\frac{1}{\Gamma(\alpha_0)}\right]\cdot\beta_0^{-q\alpha_0}(\prod_{i=1}^{q} y_i)^{(\alpha_0-1)}\exp(-\frac{T_x}{\beta_0})\theta^{-m1}\theta^{-m2}}{\int_0^X\int_0^X e^{-q}\exp(-\frac{T_x}{\theta})\left[\frac{1}{\Gamma(\alpha_0)}\right]^q\beta_0^{-q\alpha_0}(\prod_{i=1}^{q} y_i)^{(\alpha_0-1)}\exp(-\frac{T_x}{\theta})\theta^{-m1}\theta^{-m2}d\beta_0 d\theta}$$

By the cancellation, the denominator of (2-6) becomes

$$f(\theta,\beta|q,\ T_x,\ T_y\ ;\alpha)$$

$$= \int_0^\infty \int_0^\infty e^{-(q+m1)}\ \beta^{-(q\alpha_0+m2)}\exp[-(\frac{T_x}{\theta}+\frac{T_y}{\beta_0})]\ d\beta_0 d\theta$$

$$= \int_0^\infty e^{-(q+m1)}\exp(-\frac{T_x}{\theta})\ [\int_0^\infty\beta^{-(q\alpha_0+m2)}\exp[-(\frac{T_y}{\beta_0})d\beta_0]d\theta$$

Suppose $\beta_0 = 1/\alpha$, then the inner integral is evaluated as

$$\int_0^\infty\beta_0^{\ -(q\alpha_0+m2)}\exp\left(-\frac{T_y}{\beta_0}\right)d\beta_0$$

$$= \int_0^\infty \alpha^{(q\alpha_0+m_2-2)}\cdot\exp(-T_y\alpha)d\alpha$$

$$= \frac{\Gamma(q\alpha_0+m_2-1)}{(T_y)(qa_0+m_2-1)}$$

Supose $\theta = 1/\beta$, then the outer integral of (2-6) is evaluated as

$$\int_0^\infty e^{-(q+m)}\exp\left(-\frac{T_x}{\theta}\right)d\theta$$

$$= \int_0^\infty \beta^{(q+m_1-2)}\exp(-T_x\cdot\beta)d\beta$$

$$= \frac{\Gamma(q+m_1-1)}{(T_x)(q+m_1-1)} \qquad (Q,E,D)$$

LEMMA2.2 The posterior distribution of component stead-state availability $A=1/(1+\alpha_0\delta)$ is given by

$$R(A|q,\ Tx,\ Ty;\ \alpha_0)$$

$$=\left(\frac{\alpha_0 Ty}{Tx}\right)^{(qa_0+m_2-1)} A^{(qa_0+m_2-2)}\ (1-A)^{(q+m_1-2)}$$

$$= B(q+m_1-1, qa_0+m_2-1)\left[1-A\left(1-\frac{\alpha_0 Ty}{Tx}\right)\right]^{k},\ (0<A<1)$$

,where B(.,.)is Beta funtion, $k=(q+m_1+qa_0+m_2-2)$and $\delta = \beta_0/\theta$ is the service factor.

PROOF. First, we find the posterior distribution of $\delta = \beta_0/\theta$.According to **LEMMA2.1**, we express that

$$R_\delta(\delta|q,\ Tx,\ Ty;\ \alpha_0)\beta_0\delta^{-2}d\beta_0$$

$$=\int_0^\infty S(\beta_0\backslash\delta,\beta_0|q,\ Tx,\ Ty;\ \alpha_0)\beta_0\delta^{-2}d\beta_0$$

$$=\frac{(Tx)^{(q+m_1-1)}(Ty)^{(qa_0+m_2-1)}}{\Gamma(q+m_1-1)\Gamma(qa+m_2-1)}\int_0^\infty\left(\frac{\beta_0}{\delta}\right)^{-(q+m_1)}\beta^{-(qa_0+m_2)}\exp\left[-(\delta/\beta_0 Tx + 1/\beta_0 Ty)\right]\beta_0\delta^{-2}d\beta_0$$

$$=\frac{(Tx)^{(q+m_1-1)}(Ty)^{(qa_0+m_2-1)}}{\Gamma(q+m_1-1)\Gamma(qa+m_2-1)}\int_0^\infty\beta_0^{-(q+m_1+qa_0+m_{2a}-1)}\exp\left[-1/\beta_0(\delta Tx+Ty)\right]d\beta_0$$

Let $\beta_0 = 1/\alpha$, then we have integration with respect to β_0 that

$$R_\delta(\delta|q,\ Tx,\ Ty;\ \alpha)$$

$$=\frac{(Tx)^{(q+m_1-1)}\ (Ty)^{(qa_0+m_2-1)}\ \delta^{(q+m_1-2)}\ \Gamma(q+m_1+qa_0+m_2-2)}{\Gamma(q+m_1-1)\Gamma(qa_0+m_2-1)\cdot(\delta Tx+Ty)^{(q+m_1+qa_0+m_2-2)}}$$

$$=\frac{(Tx)^{(q+m_1-1)}\ (Ty)^{(qa_0+m_2-1)}\ \delta^{(q+m_2-2)}}{B(q+m_1-1,qa_0+m_2-1)(\delta Tx+Ty)^{(q+m_1+qa_0+m_2-2)}},\ 0<\delta<\infty$$

Accordingly, the posterior distribution of component steady-state availability $A = 1/(1+\alpha_0\ \delta)$ is given by

$$R(A|q,\ Tx,\ Ty;\ \alpha_0)$$

$$=R_\delta\left[\frac{(A^{-1}-1)}{\alpha_0}\Big|q,Tx,Ty;\alpha_0\right]\left(\frac{A^{-2}}{\alpha_0}\right)$$

$$=\frac{(Tx)^{(q+m_1-1)}(Ty)^{(qa_0+m_1-1)}(1-A)^{(q+m_1-2)}}{B(q+m_1-1,qa_0+m_2-1)\left[Ty+Tx(A^{-1}-1/\alpha_0\right]^{(q+m_1+qa_0+m_2-2)}}$$

(Q.E.D.)

By using **LEMMA** 2.1 and **LEMMA** 2.2, we can prove the following theorem.

THEOREM 2.1 Under a squarred-error loss function, Bayesian estimation of component steady-state availability A is given by

$$A_1^{*}=\frac{qa_0 + m_2 - 1}{q + m_1 + qa_0 + m_2 - 2}\cdot {}_2F_1(1,q+m_1-1;q+m_1+qa_0+m_2-1;1-\frac{Tya_0}{Tx})$$

,where $0 < a_0Ty/Tx < 2$, ${}_2F_1(a,b;c;t)$ is a confluent hypergometic function in Gauss form.

PROOF. Bayer estimation of component steady-state availability A is average of posterior distrbution. From (2-2) and(2-3), we redure as follow;

$$A_1^{*} = \int_0^1 A\ h(A|q,\ Tx,\ Ty;\alpha_0)dA$$

$$= \frac{\left[\frac{\alpha_0 Ty}{Tx}\right]^{(q\alpha_0+m_2-1)}}{B(q+m_1-1,q\alpha_0+m_2-1)} \int_0^1 \frac{A^{(q\alpha_0+m_2-1)}(1-A)^{(q+m_1-2)}}{\left[1-A(1-\frac{\alpha_0 Ty}{Tx})\right]^{(q+m_1+q\alpha_0+m_2-2)}}\cdot dA$$

$$= \frac{\left[\frac{\alpha_0 Ty}{Tx}\right]^{(q\alpha_0+m_2-1)}\ \Gamma(q+m_1+q\alpha_0+m_2-1)\Gamma(q\alpha_0+m_2)\Gamma(q+m^1-1)}{\Gamma(q+m_1-1)\Gamma(q\alpha_0+m_2-1)\ \Gamma(q+m_1+q\alpha_0+m_2-1)}\cdot Q1$$

$$= \frac{\left[\frac{\alpha_0 Ty}{Tx}\right]^{(q\alpha_0+m_2-1)}\cdot(q\alpha_0+m_2-1)}{q+m_1+q\alpha_0+m_2-2}\left[\frac{\alpha_0 Ty}{Tx}\right]^{-(q\alpha_0+m_2-1)}\cdot Q2$$

$$= \frac{q\alpha_0+m_2-1}{q+m_1+q\alpha_0+m_2-2}\cdot Q2$$

, where $Q1={}_2F_1(q+m_1+q\alpha_0+m_2-2;q\alpha_0+m_2;q+m_1+q\alpha_0+m_2-1;1-(\alpha_0 Ty/Tx))$
and $Q2 = \lambda F_1(1,q+m_1-1;q+m_1+q\alpha_0+m_2-1;1-(\alpha_0 Ty/Tx))$.
(Q.E.D.)

3. Bayes Estimation for Conjugate Prior Distribution.

In this section, by employing the technique similar to that of the previous section, we shall obtain Bayes estimation of steady-state availability for conjugate prior distribution. Suppose MTBF in the exponential failure time distribution and the scale paramater in Gamma repair time distribution have independent conjugate density function;

$$(3\text{-}1)\quad K_1(\theta) = \frac{b^a}{\Gamma(\theta)}(1/\theta)^{(a+1)}\ \exp(-\frac{b}{\theta}),\ \theta > 0,a,b\geq 0$$

$$(3\text{-}2)\quad K_2(\theta) = \frac{d^c}{\Gamma(c)}(1/\beta)^{(c+1)}\ \exp(-\frac{d}{\beta_0}),\ \beta_0 > 0,c,d\geq 0.$$

Then, we have the following lemma.

LEMMA 3.1 The joint posterior density function of θ and β is given by

$$K(\theta,\beta_0|q,t_x,t_y;\alpha_0) = \frac{(Tx+b)^{(q+a)}(Ty+d)^{(q\alpha_0+c)}\exp[-(1/\theta)(Tx+b)-(1/\beta_0)(Ty+d)]}{\theta^{(q+a+1)}\beta_0^{(q\alpha_0+c+1)}\cdot\ \Gamma(q+a)\Gamma(q\alpha_0+c)}$$

PROOF. From(3-1) and(3-2), the joint distribution of θ and β_0 is as follow,

$$K(\theta,\beta_0|q,T_x,T_y;\alpha_0) = \frac{L(q,Tx,Ty|\theta,\beta_0;\alpha_0)\ k_1(\theta)k_2(\beta_0)}{\int^{\infty}\!\!\int^{\infty} L(q,Tx,Ty|\theta,\beta_0;\alpha_0)\ k_1(\theta)\cdot k_2(\beta_0)\ d\beta_0\ d\theta}$$

0 0

$$= \frac{\theta^{-q}\exp(-Tx/\theta)[1/\Gamma(\alpha_0)]^q \beta_0^{-q\alpha_0}(\prod_{i=1}^{q} y_i)^{(\alpha_0-1)} \exp(-Ty/\beta_0)[(b^q\Gamma(a))\theta^{-(a+1)}]}{\int_0^\infty\int_0^\infty \theta^{-q}\exp(-\frac{Tx}{\theta})[\frac{1}{\Gamma(\alpha_0)}]^q \beta_0^{-q\alpha_0}(\prod_{i=1}^{q} y_i)^{(\alpha_0-1)} \exp(-Ty/\beta_0)(Ty/\Gamma(a))\theta^{-(a+1)}\exp(-b/\theta)}$$

$$* \frac{\exp(-b/\theta)(d^c/\Gamma(c))\beta_0^{-(c+1)}\exp(-d/\beta_0)}{(d^c/\Gamma(c))\beta_0^{(c+1)}\exp(-d/\beta_0)d\beta_0 d\theta}$$

By the cancellation

$$\int_0^\infty\int_0^\infty \theta^{-(q+a+1)}\beta^{-(a\alpha_0+c+1)}\exp[-\frac{1}{\theta}(Tx+b) - \frac{1}{\beta_0}(Ty+d)]d\beta_0 d\theta$$

$$(2\text{-}8) = \int^\infty \theta^{-(q+a+1)} \exp\left[-\frac{1}{}(Tx+b)\right]\int^\infty \beta_0^{-(q\alpha_0+c+1)}\exp\{-\frac{1}{}(Ty+d)\}d\beta_0\Big]d\theta$$

Suppose $\beta_0=1/\alpha$, then the inner integal of (2-8) as follows;

$$\int_0^\infty \beta_0^{-(q\alpha_0+c+1)}\exp[-\frac{1}{\beta_0}(Ty+d)]d\beta_0 = \int_0^\infty \alpha^{(q\alpha_0+c+1)}\exp[-(Ty+d)]d\alpha$$

$$= \frac{\Gamma(q\alpha_0+c)}{(Ty+d)^{(q\alpha_0+c)}}$$

and sippose $\theta=1/\alpha$, then the outer integral of (2-8) as

$$\int_0^\infty \theta^{-(q+a+1)}\exp[\{-\frac{1}{\theta}(Ty+b)\}]d\theta = \int_0^\infty \beta^{(q+a+1)}\exp[\{-\beta(Ty+b)\}]d\beta$$

$$= \frac{\beta^{(q+a)}}{(Ty+d)^{(q+a)}}$$

(Q.E.D.)

LEMMA 3.2 The posterior density function of component steady-state availability for $A=1/(1+\alpha;\delta)$ is given by

$$h(A|q,Tx,Ty;\alpha_0)$$

$$= \frac{(\alpha_0 Ty/Tx)^{(q\alpha_0+m_1-1)}A^{(q\alpha_0+m_2-2)}(1-A)^{(q+m_1-2)}}{B(q+m_1-1,q\alpha_0+m_2-1)[1-A(1-\alpha_0 Ty/Tx)]^{(q+m_1+q\alpha_0+m_2-2)}}$$

,where B(.) is Beta-function.

PROOF. First, according to LEMMA 3.1,

$$S(\delta|q,Tx,Ty;\alpha_0)$$

$$= \int_0^\infty K(\frac{\beta_0}{\delta},\beta_0|q,Tx,Ty;\alpha_0)\beta_0\delta^{-2}d\beta_0$$

$$= \frac{(Tx+b)^{(q+a)}(Ty+d)^{(q\alpha_0+c)}}{\Gamma(q+a)\cdot\Gamma(q\alpha_0+c)}\int_0^\infty(\frac{\beta_0}{\delta})^{-(q+a+1)}\beta_0^{-(q\alpha_0+c+1)}\exp[-\frac{\delta}{\beta_0}(Tx+b)$$

$$-\frac{1}{\beta_0}(Ty+d)]\beta_0\delta^{-2}d\beta_0$$

$$= \frac{(Tx+b)^{(q+a)}(Ty+d)^{(q\alpha_0+c)}\delta^{(q+a-1)}}{\Gamma(q+a)\cdot\Gamma(q+c)}\int_0^\infty\beta_0^{-(q+a+q\alpha_0+c+1)}$$

$$\exp[-\frac{1}{\beta_0}\{\delta(Tx+b)^1\frac{}{\beta_0}(Ty+d)\}]d\beta_0$$

,where $1/\delta=\theta/\beta_0$. Suppose $\beta_0=1/\alpha$, then

$$S(\delta|q,Tx,Ty;\alpha_0)$$

$$= \frac{(Tx+b)^{(q+a)}(Ty+d)^{(q\alpha_0+c)}\delta^{(q+a-1)}\Gamma(q+a+q\alpha_0+c)}{\Gamma(q+a)\Gamma(q\alpha_0+c)[\delta(Tx+b)+(Ty+)+(Ty+d)]^{(q+a+q\alpha_0+c)}}$$

$$= \frac{(Tx+b)^{(q+a)}(Ty+d)^{(q\alpha_0+c)}\ \delta^{(q+a-1)}}{B(q+a,q\alpha_0+c)[\delta(Tx+b)+(Ty+d)]^{(q+a+q\alpha_0+c)}}$$

Therefore, posterior distribution of steady-state availability is as follows

$$P(A|q,Tx,Ty;\alpha_0)$$

$$= S[\frac{A^{-1}-1}{\alpha_0}|q,Tx,Ty;\alpha_0](A^{-2}/\alpha_0)$$

$$= \frac{(Tx+b)^{(q+a)}(Ty+d)^{(q\alpha_0+c)}[(A^{-1}-1)/\alpha_0]^{(q+a-1)}(A^{-2}/\alpha_0)}{B(q+a,q\alpha_0+c)[\delta(Ty+d)+(A^{-1}-1)(Tx+b)/\alpha_0]^{(q+a+q\alpha_0+c)}}$$

$$= \frac{(Tx+b)^{(q+a)}(Ty+d)^{(q\alpha_0+c)}[(A^{-1}-1)/\alpha_0]^{(q+a-1)}A^{-2}/\alpha_0}{B(q+a,q\alpha_0+c)\{1-A[1-\{\alpha_0(Ty+d)/(Tx+b)c\}]\}^{(q+a+q\alpha_0+c)}}$$

$$= \frac{[\{\alpha_0(Ty+d)\}/(Tx+b)]^{(q\alpha_0+c)}\ A^{(q\alpha_0+c-1)}(1-A)^{(q+a-1)}}{B(q+a,q\alpha_0+c)[1-A[1-\{\alpha_0(Ty+d)/(Tx+b)c\}]]^{(q+a+q\alpha_0+c)}}$$

(Q.E.D.)

THEOREM 3.1 Under the squarred-error loss function, Bayes estimation steady-state availability A is given by

$$A_2^* = \frac{q\alpha_0 + c}{q + a + q\alpha_0 + c}\ {}_2F_1[1,q+a;q+a+q\alpha_0+1;1-\alpha_0(Ty+d)/(Tx+b)]$$

,where $0<(Ty+d)/(Tx+b)<2$, ${}_2F_1(a,b,;c;t)$ is a confluent hypergometric in Gauss form.

PROOF. Form **LEMMA 3.2,** (2-2) and (2-3), we have following Bayes estimation of component steady-state availability A;

$$A_2 = \int_0^1 A\ P(A|q,Tx,Ty;\alpha_0)dA$$

$$= \frac{[\alpha_0(Ty+d)/(Tx+b)]^{(q\alpha_0+c)}}{B(q+a,q\alpha_0+)}\int_0^1 \frac{A^{(q\alpha_0+c)}(1-A)^{(q+a-1)}}{[1-A\{1-\alpha_0(Ty+d)/(Tx+b)\}]^{(q+a+q\alpha_0+c)}}dA$$

$$= \frac{[\alpha_0(Ty+d)/(Tx+b)]^{(q\alpha_0+c)}\ \Gamma(q+a+q\alpha_0+c)\Gamma(q\alpha_0+c+1)\ \Gamma(q+a)}{\Gamma(q+a)\Gamma(q\alpha_0+c)\ \Gamma(q+a+q\alpha_0+c+1)}*K1$$

$$= \frac{[\alpha_0(Ty+d)/(Tx+b)]^{(q\alpha_0+c)}(q\alpha_0+c)}{(q+a+q\alpha_0+c)}[\alpha_0(Ty+d)/(Tx+b)]^{-(q\alpha_0+c)}\cdot K2$$

$$= \frac{q\alpha_0+c}{q+a+q\alpha_0+c}\cdot K2$$

where $K1={}_2F_1[q+a+q\alpha_0+c,q\alpha_0+c+1;q+a+q\alpha_0+c+1;1-\alpha_0(Ty+d)/(Tx+b)]$,

and $K2 = F[1,q+a;q+a+q\alpha_0+c+1;1-\alpha_0(Ty+d)/(Tx+b)]$. (Q.E.D.)

4.Numerical results

In this section, several numerical estimations were compared with each other. As is seen from Table 4.3 the estimations nearest to true value considered as those A_1* and A_2*· which we reduced from the above sections.

Example

Cycle	Failure times	Repair times
1	725.67	18.34
2	280.04	16.84
3	850.58	13.69

4	845.76	17.83
5	195.10	16.76
6	732.36	15.78
7	528.40	20.96
8	610.12	18.83
9	327.84	19.73
10	310.12	17.84

Table 4.1

$q=5, \theta=500, \alpha_0=3, \beta_0=5$

(m_1, m_2)	(1,2)	(3,5)	(5,9)	(4,8)
True A	A_1^*	A_1^*	A_1^*	A_1^*
0.971	0.966	0.863	0.957	0.943

Table 4.2

$q=5, \theta=500, \alpha_0=3, \beta_0=5$

(a,b)(a,d)	(1.5)(1,5)	(2.10)(3,7)	(2,5)(3,4)	(3,3)(4,8)
True A	A_2^*	A_2^*	A_2^*	A_2^*
0.971	0.985	0.962	0.957	0.952

Table 4.3

True A	A_1^*	A_2^*	MLE
0.971	0.863	0.962	0.967

where, $A_1^*(3,5)$ $A_2^*(2,10),(3,7)$

REFERENCES

(1).Brender, D.M. The Prediction and Mearsurement of System availability; Advanced Application and Technique, IEEE Transaction of Reliability,vol.17 (1968)138-147.

(2).Erdelyi,A;Higher Transcendental Function, vol.1 Mc-Grow Hill, N.Y.1953.

(3).Gaver,D.P.and Mazumder, M;Statistical Estimation in a problem of System Reliability, Naval Research Logistic Quarterly, vol.14, (1967)473-488.

(4).Nelson, W;A Statistical Prediction Interval of Availaility, IEEE Transaction on Reliability, vol.19,(1970)179-182.

(5).Thompson, W.E and Palicio, P.A;Baysian Confidence Limites for the Availability of System, IEEE transation of Reliability, vol.24(1975)118-120.

(6).Rupert, G.and Miller Jr.;Survival Analysis, John Willy and Sons, 1981.