A Study on the Usefulness of Linear Notch Mechanics

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요 약

균열(Crack)을 갖는 部材의 강도는 가혹함(Severity)의 척도로서 응력확대계수(Stress intensity factor)를 이용하는 선형파괴역학(선형균열역학)의 개념을 이용하여 예측된다. 한편, 노치(Notch)를 갖는 部材의 강도는 가혹함의 척도로서 최대응력과 노치반경을 이용하는 선형노치역학의 개념을 이용하여 예측된다. 이하에서는 선형파괴역학과 선형노치역학이 생겨나게 된 역사적 배경을 간단히 언급하고, 선형노치역학의 개념에 의해 설명되는 제반현상과 그 유용성에 대해 해설하기로 한다.

1. Introduction

The purpose of the research concerning strength of materials is to predict the behavior of real objects from the behavior of specimens by the simplest procedures. To accomplish this purpose, it is necessary to know two items : one is the severity controlling the failure or fracture and is the mechanics behavior of the given materials.

This parper is concerned with the former.

In most cases, the failure or fracture of mechanics and structures are brought by the stress concentration due to the existence of a crack or a notch. On the other other hand, it is well known that the elastic maximum stress is not enough to predict the failure or fracture. Therefore, it is very important to evaluate the severity of the part having a crack or a notch.

Concerning a crack, linear fracture mechanics[1] can play an important role to predict the fracture or yield in neighborhood of a crack tip. That is, stress intesity factor is the effective measure of severity controlling the failure or fracture in a cracked body. However, concerning a notch, concept corresponding to the linear fracture mechanics is not discussed fully until now.

In general a crack appears necessarily before any kinds of fracture. Therefore, it may be considered that fracture mechanics is sufficient in treating all kinds of fracture problems. However, many notch problems cannot be treated correctly without considering the characteristics of notches. For example, the problem of ductile－brittle transition due to notch sharpness in tensile tests of polycarbonate notched specimens[2] and the problem of non－propagationg fatigue crack in sharply notched specimens[3] cannot be explained from the stand－point of fracture mechanics.

Originally, linear fracture mechanics should be called as linear crack mechanics. If we call linear fracture mechanics as linear crack mechanics, we will notice easily that linear notch mechanics is necessary.

Linear notch mechanics[4] is defined as an engineering method which treats the notch problems by elastic stress fields alone, in the same way that the linear crack mechanics is defined as an engi - neering method which treats the crack problems by elastic stress fields(stress intensity factors)alone.

In this paper, the theoretical background of linear notch mechanics will be given comparing the case of a notch with the case of crack and the usefulness of linear notch mechanics will be shown thrrough some examples. The short history of linear crack mechanics(linear fracture mechanics) and linear notch mechanics will slao be given.

2. Necessity of the measure of severity in a crack or a notch

When there are the origins for stress concentration, in general the elastic maximum stress σ_{max} is not a unique parameter controlling failure[5,6], That is, the same phenomena can be seen in the different values of σ_{max}.

According the Griffith's paper[7], the phenomena seen in reference[5] are the starting point of linear crack mechanics(linear fracture mechanics). Similarly, the phenomena seen in reference[6] are the starting point of linear notch mechanics, as shown later.

From reference[5], it can be concluded that the measure of severity in a crack or a notch is necessary in order to predict the behavior of real objects from the behavior of specimens. Table 1 shows the procedure for predicting the behavior of real objects having a notch. In case of a crack, the procedure based on stress intensity factor is used, as well known.

Table 1. Method for predicting the behavior real objects having a notch from the behavior of specimens.

When the same phenomena occur under $\sigma_{max1}=\sigma_{max2}$(special case) :	
a) Stress of specimen b) Behavior of specimen c) Stress of real objects	⇒ Behavior of real objects prediction
When the same phenomena does not occur under $\sigma_{max1}=\sigma_{max2}$(general case) :	
a) Stress of specimen b) Behavior of specimen c) Stress of real objects d) Measure of severity	⇒ Behavior of real objects prediction

3. Characteristics of stress filed due to a crack or a notch

When an infinite plate with an elliptic hole is subjected to uniform tension at infinity, the elastic stress distribution on the x - axis is given by the following equation[3].

$$\sigma_y(x) = \frac{m^4\xi^3 + m^2(m^2+m-3)\xi + (m+1)}{(m-1)(m^2-1)} \qquad (1)$$

Where

$$m = \sqrt{\frac{a}{\rho}}$$

$$\xi = \frac{a+x}{a\rho + 2ax + x^2}\sqrt{\frac{\rho}{a}} \qquad (2)$$

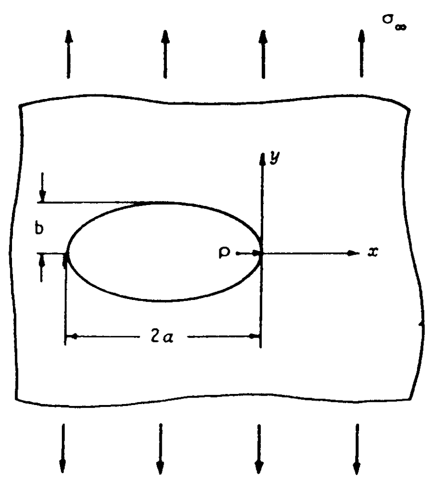


Fig. 1. Notations in Equation(1) and (2)

Notations are given in Figure1.

Based on Equation (1), both characteristics of elastic stress fields due to a crack and a notch are obtained below.

3.1 Stress field near the tip of a crack

A crack is considered as a notch flattened extremely. Thus putting $\rho \rightarrow 0$ in Equations(1) and (2) and considering $x/a \ll 1$, elastic stress distribution on the x – axis near the tip of a crack is obtained as follows :

$$\rho_y(X) = \frac{\sigma_{\infty}\sqrt{\pi a}}{\sqrt{2\pi x}} = \frac{K_1}{\sqrt{2\pi x}} \qquad (3)$$

Where $K_1 = \sigma_{\infty}\sqrt{\pi a}\ \infty$ (4)

K_I is the stress intensity factor for mode I loading. Equation(3) means that the elastic stress distribution on the x – axis $\sigma_y(x)$ is determined mainly by K_I alone.

(i) Same elastic stree fileds : When the value of K_I are constant, the elastic stress distribution near the tips of cracks are equal to each other, irrespective of crack size or the other geometrical conditions of cracked bodies[1], Figure 2(a).

(ii) Same responses : The additional stress fields due to a given amount of plastic deformation occurring at a given place near the crack tips are equal to each other, irrespective of cracked size or the other geometrical conditions of cracked bodies[4], Figure 2(b).

The same response is due to geometrical similarity near the crack tips as seen by the circle in Figure 2(b). The same responses can be cracked by direct calculation based on the fact that plastic deformations can be replaced by pairs of point forces acting in the perfect elastic body[8].

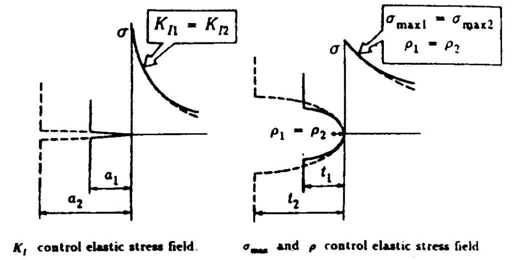
From the above two facts (i) and (ii), the following conclusion is obtained.

If the external loads are adjusted in order that K_I values are equal in the two members with a crack, the elastic – plastic stress distributions in both members become equal to each other even after the materials near the crack tips undergo slight plastic deformations.

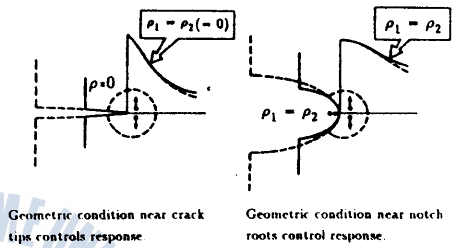
The definition of small scale yielding has to be checked based on the concept of the smae responese.

The smae elastic – plastic stress fields should result in the same phenomena.

Accordingly, the same K_I values under small scale yielding should result in the same phenomena.



(a) Condition for the same elastic stress ficlds in two cracked bodies or two notched bodies.



(b) Condition for the same responses against plastic deformation in two cracked bodies or two notched bodies.

Fig. 2 The same elastic stress fields and the same responses in two cracks or two notches.

3.2 Stress field near the root of a notch

When $x/\rho \ll 1$ in Equations (1) and (2), the elastic stress distribution near an elliptic hole is given as follows :

$$\sigma_y(X) = \sigma_{max}\{1 - \frac{3+4m}{1+2m}(\frac{x}{\rho}) + \frac{2}{3}\cdot\frac{5+6m}{1+2m}\frac{x^2}{\rho} + \cdots\} \quad (5)$$

Where $\sigma_{max} = K_t\ \sigma = (1+2)\sigma$

In Equation(5), the variations of coefficient of x/ρ and (x/ρ)2 due to the change in the values of m are small. For example, the value of (3+4m)/(1+2m) changes only from 3 to 2, according as m changes from 0 to ∞

Therefore, it is found from Equation(5) that when x/ρ is small the elastic stress distribution near the notch root is determined mainly by the maximum elastic stress ρ_{max} and ρ alone.

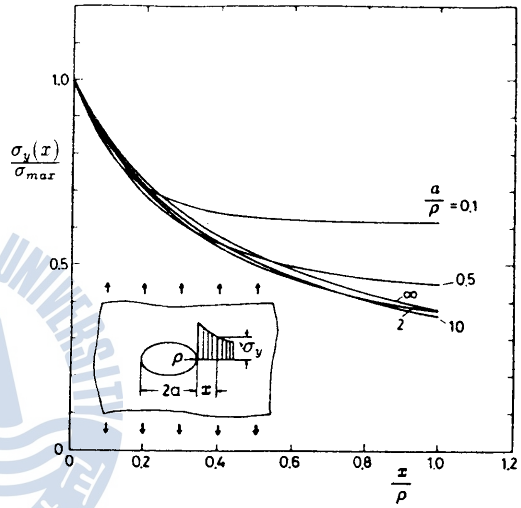
Figure 3(a) demonstrates that the elastic stress distribution on the x - axis near an elliptic hole is determined by σ_{max} and ρ alone, as long as the notch depth is not extremely shallow. Schijve has shown that not only $\sigma_y(x)$ but also the whole stress field near the notch root is determined by σ_{max} and ρ alone[9].

Although above discussion is limited to the elliptic hole as an example of notches, this significant situation holds similarly in the infinitely deep hyperboloidal notches subjected to tension or bending, Figure 3(b), and in the round bars having a circumferential notch subjected to tension or bending[10,11].

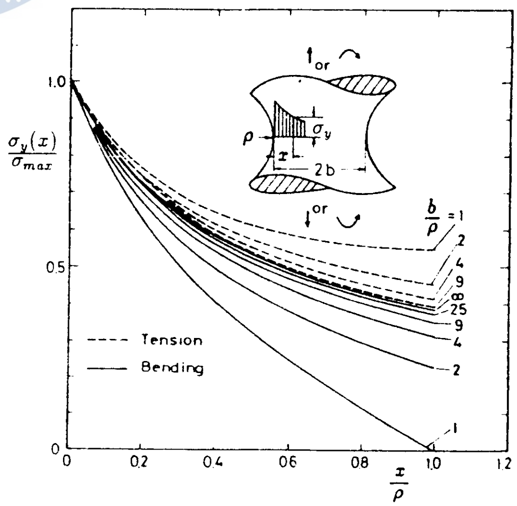
An analogous situation as in crack problems exists in notch problems : that is, if the condition of small scale yielding is satisfied the load necessary for causing fracture or a given size of plastic zone is controlled by σ_{max} and ρ alone. This is based on the following two facts similarly as in the case of crack problems(4).

(i) Same elastic stress fields : When the values of σ_{max} and ρ are constant respectively in various notches, the elastic stress distributions near the notch roots are nearly equal to each other, irrespective of notch depth or the other geometrical conditions of notched bodies[3], as shown in Figure 2(a).

(ii) Same responses : In the case where ρ is constant, the additional stress fields due to a given amount of plastic deformations occurring at a given place near the notch roots are nearly equal to each other, irrespective of notch depth or the other geometrical conditions of notch depth or the other geometrical conditions of



(a) Elliptic hole



(b) Hyperboloidal notch

Fig. 3 Elastic stress fields near a notch root are mainly determined by the values of σmax and ρ alone.

notched bodies[4], as shown in Figure 2(b).

The same responses for the case of notches can also be checked by direct calculation, similarly as in the case of cracks.

Due to the above two facts(i) and (ii), if the external loads are adjusted in order that σ_{max} values are equal in the two members having a notch with the same size of ρ, the elastic – plastic stress distributions in both members become equal to each other even after the materials near the notch roots undergo slight plastic deformations.

The same elastic – plastic stress fields should result in the same phenomean. Accordingly the same respective values of σ_{max} and ρ under small scale yielding should result in the same phenomena in notched members, as shown in Figure 4.

The similarity of elastic – plastic stress fields after a small plastic deformation is assured by the facts(i) and (ii) in both cases of crack and notch. As the measure of the severity of stress field, therefore, we should use σ_{max} and ρ for notch problems as well as K_I for crack problems.

We call here the concept, treating the strength of cracked members by the use of stress intensity factors, as linear crack mechanics and the concept, treating the strength of notched members by the use of ρ_{max} and ρ, as linear notch mechanics. Linear crack

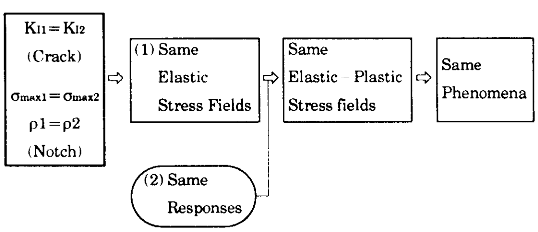


Fig. 4 The concepts of linear crack mechanics and linear notch mechanics(Condition for occurring the same phenomena in two cracked bodies or two notched bodies).

mechanics of course means linear fracture mechanics. Both concepts of linear notch mechanics and linear crack mechanics are based on the similarity of elastic stress fields and the same responses against small plastic deformations and stand on a common physical basis.

Based on linear crack mechanics and linear notch mechanics, the condition for causing the same phenomena in two cracked bodies or two notched bodies is obtained, as shown in Figure 4. In the figure, only the case of mode I is treated, because the cases of the other mode are similar to the case of mode I.

In plate problems, the similarity of stress fields and the same responses are assured only when the thickness of the plates is the same. The effect of plate thickness in fracture should be discussed from this point of view.

5. Short history of linear crack mechanics and linear notch mechanics

Griffith treated the problem of brittle fracture in his famous paper[7] published in 1921, however the original purpose of his research was to investigate the effect of surface scratches on the fatigue strength of structural members. He supposed that the K_t value of typical surface scratches is about 2～6, inferring from the result of his experimental stress analysis and the K_t value of an elliptic hole[12] in an infinite plate subjected to tension. On the other hand, the references at that time showed that the rates of reduction in fatigue limit due to the surface scratches were below about 20%.

From these facts he concluded that the elastic maximum stress at one point is not enough to predict fracture. so, he introduced the concept to energy balance and treated by the concept the brittle fracture of glass with a crack.

In the concept, the mechanical severity is defined as the energy release rate which is a kind of mean stress field near a crack tip.

Irwin treated again the stress field itself directlu[13], probably supposing that the driving force for crack extension exists in the neighborhood of a crack tip alone. He showed that Griffith's energy release rate obtained by integrating the strain energy of all the plate can be obtained from the information at the crack tip alone. This means that the mechanical severity in the brittle fracture under tension, the energy release rate, can be represented by the stress intensity factor K_I alone. Because K_I is a parameter standing for strength of stress filed, K_I is effective for many fracture problems other than brittle fracture in the members haing a notch.

In that case, we must use linear notch mechanics.

In 1939, Ono clarified that the fatiue limit of a steel specimen with a hyperboloidal circumferential notch(Neubertype specimen) is not determined by the elastic maximum stress, σ_{max}, alone, by using his original rotating bending fatigue testing machine[14], For example, the fatigue limit of the notched specimen whose I_t is 5 was about helf of that of the unnotched specimen(Figure 5). This means that the elastic maximum stress at one point is not enough to predict the fatigue fracture of notched specimens.

Considering this fact, is 1948 Isibasi, the successor of one, proposed the 2nd point theory[15], in which the local stress in the vicinity of a small finite distance, ε_0, normal to the surface of a notch root is used as the measure of severity. He applied successfully the theory to the problems of local yielding in the plate specimen with a hole.

In 1951, Isibasi discover the nonpropagating

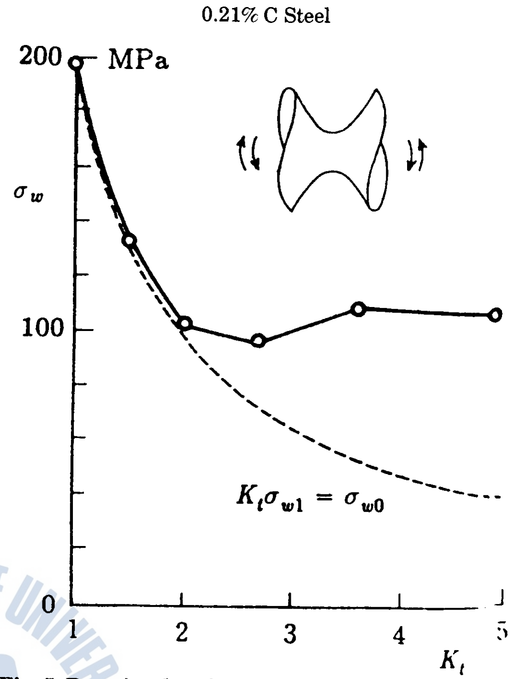


Fig. 5 Rotating bending fatigue tests Neuber type notched specimen by Ono(14). This shows that σ_{max} is not a unique parameter controlling fatigue fractures.

crack in the rotating bending fatigue test of the Neuber - type notched steel specimen[16]. At almost the same period, Lessells et al[17]. also observed the non - propagating crack, but Isibasi was the first researcher who discovered the branch point, at which the non - propagating crack appears, and clarified that the fatigue limit of notched specimens is composed of the following two limiting stress[18](Figure 6).

One is limiting stress, σ_{w1}, for crack initiation. The other is the limiting stress, σ_{w2}, for crack propagation in range where the non - propagating crack can exist, Both σ_{w1} and σ_{w2} are nominal stresses.

In 1968. Nisitani, the successor of Isibasi, showed experimentally that limiting condition for the existence of non - propagating fatigue crack is controlled by the value of notch root radius ρ alone, of the specimen having circumferential V – notch in the rotating bending fatigue test. That is, he showed that value of ρ becomes constant at the branch point(the critical point at which the non – propagating crack appears) is a given material, irrespective of the notch depth or the diameter at the minimum section[3](Figure 7). In the process analyzing the reason for this, he found that the elastic stress field near the notch root of the following notches is determined mainly σ_{max} and ρ alone[3] : they are an elliptic hole in an infinite plate subjected to tension and an infinetely deep hyperboloidal notch in an infinite body subjected to tension or bending(Figure 3). This was also confirmed for the round bar having a circumferential notch subjected to tension or bending[10,11]. Therefore, if σ_{max} and ρ are give, the elastic stress fields near a notch root is substantially controlled by σ_{max} and ρ anone[3]. Though in the original paper[3] σ_{w1} was connected to the relative stress gradient at the notch root and σ_{w2} was regarded as nearly constant in the range where $\sigma < \sigma_0$(the value of ρ at the branch point), practically these situation corresponds to the above statement in the con-

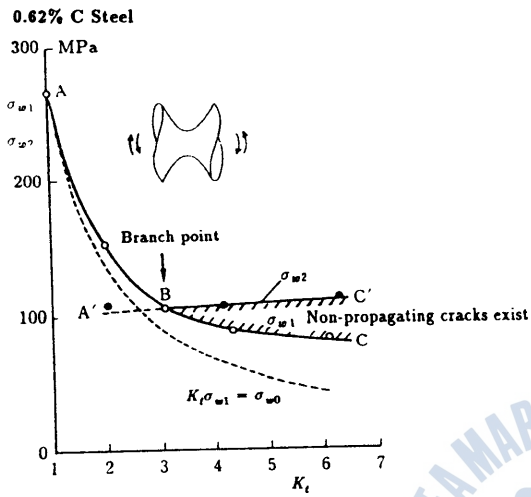


Fig. 6 Isibasi clarified that the fatigue limit of notched specimens is composed of the following two kinds of stresses : the limiting stress for crack initiation, σ_{w1}, and the limiting stress for fracture in the range where non – propagating cracks exist, σ_{w2}[18]

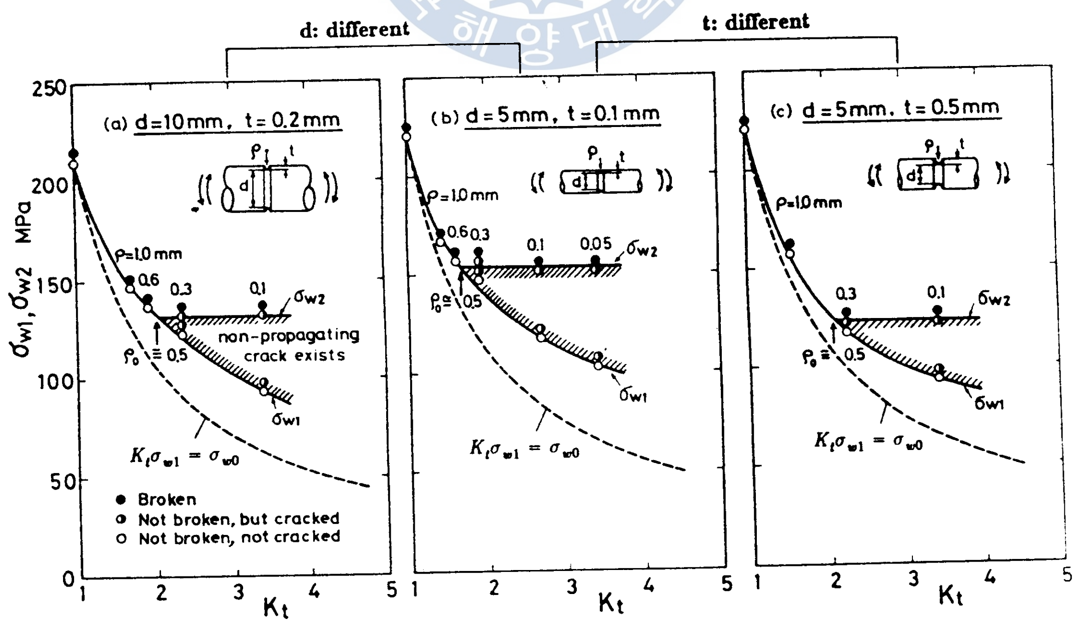


Fig. 7 Nisitani showed that value of ρ at the branch point are($\rho = \rho_0$) in rotating bending fatigue tests, independently of the notch depth or the diameter at the minimum section(3).

Table 2. Short history of linear crack mechanics and linear notch mechanics(We can the close between the two concepts).

Linear crack mechanics	Linear notch mechanics[4,19]
Motivation : σ_{max} is not enough in fatigue of surface scratched specimens[7] specimens[14]	Motivation : σ_{max} is not enough in fatigue of noched
⇩	⇩
Because σ_{max} at a point is not enough, a kind of mean was considered : Treated problem is brittle fracture of cracked glass specimens[7]	Because σ_{max} at a point is not enough, a kind of mean was considered : Treated problem is yielding of notched steel specimens[15]
⇩	⇩
Severity was treated by elastic stress field itself : Measure of severity is K[1]	Severity was treated by elastic stress field itself : Measure of severity is σ_{max} and ρ[3,4]
⇩	⇩
Application to various problems	Application to various problems

⇧ ⇧

Background of both concepts[4]

1) The same elastic stress fields

2) The same responses

tent.

In 1983[4], Nisitani, used first the word linear notch mechanics and formulated the concept of linear notch mechanics in the form shown in Figure 4[9].

Table 2 shows schematically the short history of linear crack mechanics and the similarity between the two concepts.

6. Usefulness of linear notch mechanics

The usefulness of linear crack mechanics has been recongnized fully until now. Therefore, in this section the usefulness of linear notch mechanics alone will be shown through three examples.

6.1 Condition for causing the same plastic zones in notched plates

The first example is concerned with plastic zone in the case of tension of semi - infinite plates having a semi - elliptic notch(20). Figure

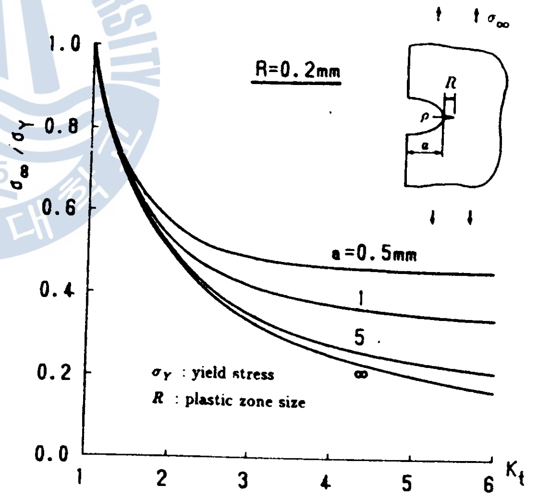


Fig. 8 Relation between the stress for causing the same plastic zones σ_∞ and the stress concentration factor K_t(Tension of a semi - infinite plate with a semi - elliptic notch)

8 shows the relation between the stress σ_∞ which produce the given size of plastic zone based on the dugdale model and the stress concentration factor K_t. In the figure a means notch depth. There is no one to one correspondence between σ_∞ and K_t. According to the con-

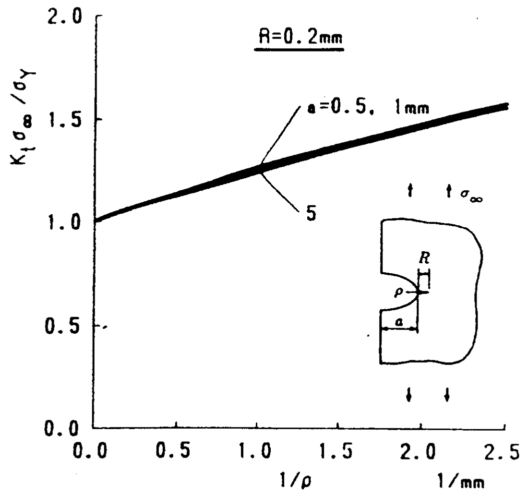


Fig. 9 Relation between $K_t\sigma_\infty$ and 1/ρ. obtained from the data in fig 10.

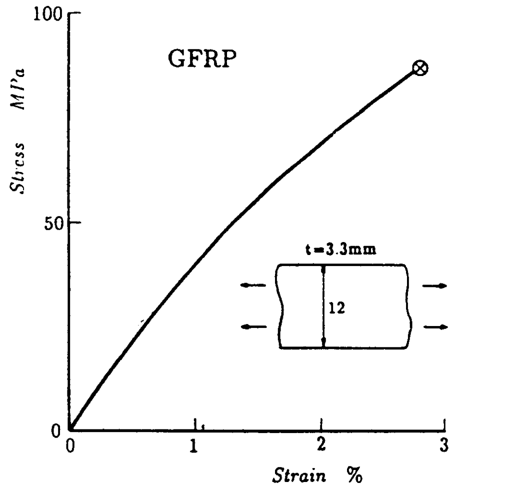


Fig. 10 Stress－strain relation of the GFRP plain specimen

cept of linear notch mechanics, there should be one to one correspondence between K_t σ_∞ and 1/ρ, independently of the value of notch depth a. Figure 9 proves it, that is, the relation between K_t σ_∞ and 1/ρ is almost independent of notch depth a.

6.2 Condition for causing brittle fracture in notched plates

The second example is concerned with brittle fracture in the case of tension of GFRP plate specimen having an internal notch[20], Figure 10 shows the stress－strain relation of the GFRP[20], Figure 11 shows the relation between the stress causing brittle fracture σ_s and the stress concentration factor K_t, in the plate specimens having various notch depths and notch root radii. There should be one to one correspondence between $\sigma_s$and K_t. According to the concept of linear notch mechanics, there should be no one to one correspondence between $K_t\sigma_s$ and 1/ρ, independently of the value of notch depth a and the other geometrical conditions. Figure 12 supports it clearly. By using Figure 12 as the

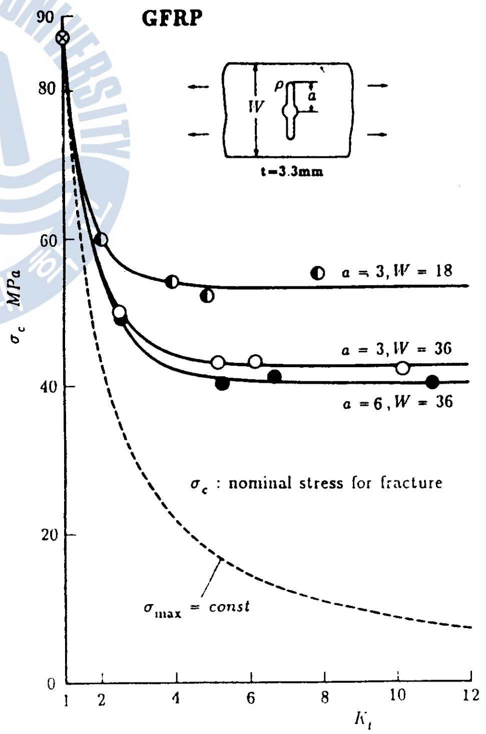


Fig. 11 Relation between the stress for causing brittle fracture σ_c and the stress concentration factor K_t(Tension of GFRP plate specimen with internal notch).

master curve for the brittle fracture of material, we can predict the limiting stress for brittle

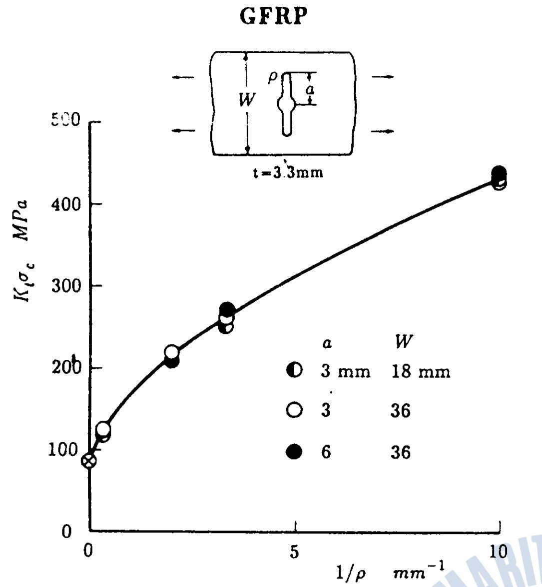


Fig. 12 Relation between $K_t\sigma_\infty$ and 1/ρ, obtained from the data in Figure 11.

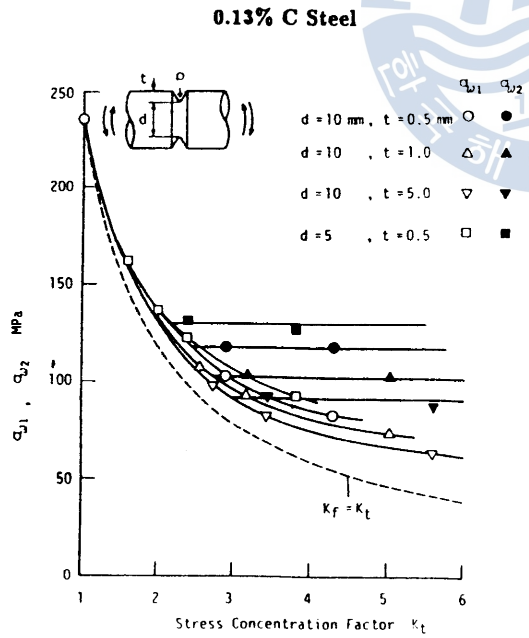


Fig. 13 Relation between the limiting stress σ_{w1} and σ_{w2}, and con - centration factor K_t. (Rotating bending of steel round bar specimen with a circum - ferential notch).

fracture on an arbitrarily sharped plate specimen habing the same thickness. This proves the usefulness of linear crack mechanics.

6.3 Condition for causing fatigue fracture in notched plates

The final example is concerned with the fatigue fracture in the case of rotating bending of steel specimens having a circumferential notch[21]. Figure 13 shows the relation between the limiting stress σ_{wl}, σ_{w2} and stress concentration factor K_t, σ_{wl} is the limiting stress for crack and σ_{w2} is limiting stress for final fracture in the range of non - propagating crack existing, In the range of non - propagating crack, σ_{wl} is equal to the limiting stress for fracture.

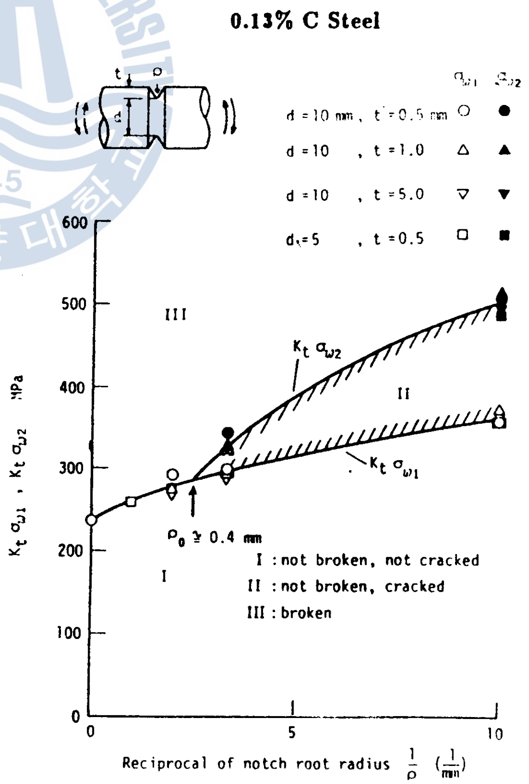


Fig. 14 Relation between $K_t\sigma_{wl}$, $K_t\sigma_{w2}$ and 1/ρ, obtained from the data in Figure 13.

There is no one to one correspondence between σ_{wl} and K_t or σ_{w2} and K_t. According to the concept of linear notch mechanics, there should be one to one correspondence between $K_t\sigma_{wl}$ and $1/\rho$ or $K_t\sigma_{w2}$ and $1/\rho$, independently of notch depth t and the diameter d. Figure 14 supports it clearly. By using Figure 14 as master curve for the fatigue strength of this material, we can predict the limiting stresses and σ_{w1} and σ_{w2} of the round bar specimen having an arbitary circumferential notch and an arbitary diameter. This proves the usefulness of linear notch mechanics.

7. Conclusions

1) According to the concept of linear notch mechanics, the measure of severity controlling the failure or fracture in a notoched body is the maximum elastic stress σ_{max} and the notch root radius ρ, in the same way that the measure of severity in a cracked body is the stress intensity factor K_I.

2) The reason why the purely elastic parameter(σ_{max} and ρ in linear crack mechanics and K_I in linear notch mechanics) is useful even for the fracture problems accompanied by plastic deformation is that the condition for the same elastic stress fields(they are bought by the same elastic parameters) includes the condition for the same responses against plastic deformation. That is, the same elastic parameters assure the same elastic stress fields and the same responses, and therefore same elastic-plastic stress fields, and finally the occurrence of the same phenomena(Figure 4).

3) The usefulness of linear notch mechanics was proved by three examples.

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