A Study on the Adaptive Controller with Fuzzy Compensator for Nonlinear Time-Varying Systems

Ko-Dong Park1), Jong-Hwa Kim2)

비선형시변시스템을 위한 퍼지보상기를 가진 적응제어기에 관한 연구

박 거 동, 김 종 화

ABSTRACT

본 논문에서는 외란을 가지는 비선형 시변시스템의 강인성과 성능 향상을 위해 새로운 적응제어 구조를 전개하였다. 표준 기준모델 적응제어기를 감독하는 역할을 하는 퍼지 보상기를 사용합으로써 성능을 크게 향상시켰고, 퍼지보상기가 안정하게 설계되었을 때 퍼지보상기가 출력오차를 유한하게 하기 때문에 표준 기준모델 적응제어기의 파라미터와 모든 신호를 유한하게 합으로써 안정도를 보장하였다. 그리고, 시뮬레이션을 수행합으로써 비선형 시변시스템인 경우 퍼지보상기를 가진 작용제어기의 유용성을 확인하였다.

I. INTRODUCTION

When uncertain parameters are contained in a controlled system or parameters are unknown, it is very difficult to design a suitable controller using conventional control methods which necessitate complete mathematical model, because of the deficient model information. In order to overcome this problem a new method named as adaptive control appeared. Adaptive control is the method which updates control parameters used to generate control input in a way that the output of unknown plant follows that of reference model so that output error goes to zero asymptotically.

Recently, new adaptive control methods were developed and their interest was concentrated in performance improvement such as transient response and steady-state response. Since the standard model reference adaptive control(MRAC) schemes have bad transient behavior when bounded disturbances are present, a modified MRAC schemes had been proposed to improve transient response. Zhihua Qu et al. proposed model reference robust control(MRRC) scheme.

¹⁾ Dept. of Control and Instrumentation Engineering, Korea Maritime University.

²⁾ Associate Professor, Dept. of Control and Instrumentation Engineering, Korea Maritime University.

robust control into MRAC in order to cope with uncertainties. However, since these schemes have been developed through mathematical modification and structural modification in standard structure, there remain some problems such as mathematical complexity and difficulty in the proof of stability, and no simplicity in applications. Especially in case when plant is modeled as nonlinear and/or time-varying system, they cannot be applied in order to assure global stability and improve performance due to mathematical restrictions.

Therefore, in this paper, a new adaptive control structure is developed and suggested in order to improve transient response and assure robustness against nonlinearity and time-varying characteristics of plant under disturbances. The structure is only the form of standard MRAC with fuzzy compensator which forces output error of MRAC to converge to zero fastly. Through simulation studies, the effectiveness of suggested MRACF is assured.

II. THE STANDARD MODEL REFERENCE ADAPTIVE CONTROL

2.1 The structure of the adaptive system

1) The controller structure

The basic structure of the adaptive system is shown in Fig. 2.1.

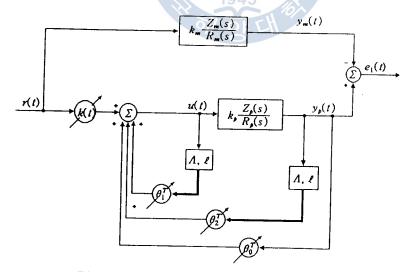


Fig. 2.1 The basic structure of the adaptive system

The controller is described completely by the differential equation

$$\dot{\omega}_1(t) = \Lambda \omega_1(t) + \ell u(t)$$

$$\dot{\omega}_2(t) = \Lambda \omega_2(t) + \ell y_p(t)$$



$$\omega(t) \triangleq [\gamma(t), \omega_1^T(t), y_p(t), \omega_2^T(t)]^T$$

$$\theta(t) \triangleq [k(t), \theta_1^T(t), \theta_0(t), \theta_2^T(t)]^T$$

$$u(t) = \theta^T(t)\omega(t)$$
(2.1)

where $k: \mathbb{R}^+ \to \mathbb{R}$, $\theta_1, \omega_1: \mathbb{R}^+ \to \mathbb{R}^{n-1}$, $\theta_0: \mathbb{R}^+ \to \mathbb{R}$, $\theta_2, \omega_2: \mathbb{R}^+ \to \mathbb{R}^{n-1}$. Λ is an $(n-1)\times (n-1)$ stable matrix and $\det[sI-\Lambda] = \lambda(s)$.

The overall system can also be represented as

$$\begin{bmatrix} \dot{x}_{p} \\ \dot{\omega}_{1} \\ \dot{\omega}_{2} \end{bmatrix} = \begin{bmatrix} A_{p} & 0 & 0 \\ 0 & A & 0 \\ \ell & h_{p}^{T} & 0 & A \end{bmatrix} \begin{bmatrix} x_{p} \\ \omega_{1} \\ \omega_{2} \end{bmatrix} + \begin{bmatrix} b_{p} \\ \ell \\ 0 \end{bmatrix} [\theta^{T}(t)\omega(t)]$$

$$y_{p} = h_{p}^{T}x_{p}.$$
(2.2)

We define the following parameter errors

$$\begin{aligned} & \psi(t) \triangleq k(t) - k^*, \ \phi_0(t) \triangleq \theta_0(t) - \theta_0^*, \ \phi_1(t) = \theta_1(t) - \theta_1^* \\ & \phi_2(t) \triangleq \theta_2(t) - \theta_2^*, \ \phi(t) \triangleq \left[\psi(t), \phi_1^T(t), \phi_0(t), \phi_2^T(t) \right]^T. \end{aligned}$$

Then the state equation (2.2) can also be written as

$$x = A_c x + b_c [k^* r + \phi^T \omega]; \quad y_p = h_c^T x$$
 (2.3)

where $x = [x_p^T, \omega_1^T, \omega_2^T]^T$, $h_c = [h_p^T, 0, 0]^T$

$$A_{c} = \begin{bmatrix} A_{p} + \theta_{0}^{\star} b_{p} h_{p}^{T} & b_{p} \theta_{1}^{\star^{T}} & b_{p} \theta_{2}^{\star^{T}} \\ \ell & \theta_{0}^{\star} h_{p}^{T} & \Lambda + \ell & \theta_{1}^{\star^{T}} & \ell & \theta_{2}^{\star^{T}} \\ \ell & h_{p}^{T} & 0 & \Lambda \end{bmatrix}, b_{c} = \begin{bmatrix} b_{p} \\ \ell \\ 0 \end{bmatrix}$$

$$(2.4)$$

When $\phi(t) \equiv 0$ that is $\theta(t) = \theta^*$, (2.3) also represents the reference model which can be described by the (3n-2)th order differential equation

$$x_{mc} = A_c x_{mc} + b_c k^* r; \quad y_m = h_c^T x_{mc}$$
 (2.5)

where $x_{mc} = [x_p^{*^T}, \omega_1^{*^T}, \omega_2^{*^T}]^T$, $h_c^T(sI - A_c)^{-1}b_c = \frac{k_p}{k_m}W_m(s)$

2) The error equation

The error equation between model and plant may be expressed as

$$\dot{e}(t) = A_c e(t) + b_c \left[\phi^T(t)\omega(t)\right]$$

$$e_1(t) = h_c^T e(t)$$
(2.6)

where $e(t) \triangleq x(t) - x_{mc}(t)$ and $e_1 = y_p - y_m$. The output error e_1 is given by

$$e_1(t) = \frac{k_p}{k_m} W_m(s) \phi^T(t) \omega(t)$$
 (2.7)



2.2 The control problem

1) Relative degree $n^* = 1$

Because a model can be chosen which has a strictly positive real transfer function

$$W_m(s) = h^T (sI - A_m)^{-1} b_m = k_m \frac{Z_m(s)}{R_m(s)}$$

the parameter error vector $\phi(t)$ is updated according to the control law

$$\dot{\phi} = \dot{\theta} = -sgn(k_p)e_1(t)\omega(t) \tag{2.8}$$

By lemma^[3], the state error e(t) and parameter error $\phi(t)$ are bounded. Since e_1 as well as the output of the reference model are bounded, y_p is bounded and $\omega(t)$ is bounded so that $e(t) \to 0$ as $t \to \infty$.

2) Relative degree $n^* \ge 2$

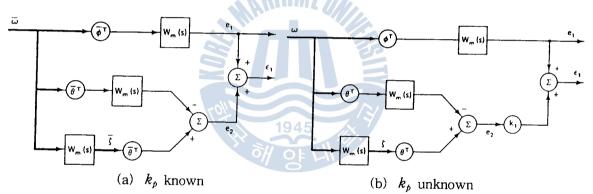


Fig. 2.2 The augmented error

case (i) k_p known

The augmented error method suggested by Monopoli^[3] can suitably modify error equation in order to implement stable adaptive laws as shown in Fig. 2.2(a). The augmented error can now be expressed as

$$\varepsilon_1(t) = \overline{\phi}^T \overline{\zeta}(t) + \delta_1(t), \quad \delta_1(t) = \overline{\theta^*}^T \overline{\zeta}(t) - W_m(s) \overline{\theta^*}^T \overline{\omega}(t)$$
 (2.9)

where $\overline{\zeta}(t) \triangleq W_m(s) I \overline{\omega}(t)$, $\overline{\phi} = \overline{\theta} - \overline{\theta}^*$, $\overline{\theta}^{*T} = [\theta_1^{*T}, \theta_0^*, \theta_2^{*T}]$, $\overline{\theta}^T \triangleq [\theta_1^T, \theta_0, \theta_2^T]$,

 $\overline{\omega}^T \triangleq [\omega_1^T, y_p, \omega_2^T]$ and $\delta_1(t)$ is an exponentially decaying signal due to initial conditions. The adaptive law having the form

$$\overline{\phi}(t) = -\varepsilon_1(t)\,\overline{\zeta}(t) \tag{2.10}$$

would suffice to assure stability. [3]



case (ii) k_p unknown

Since the feedforward gain k(t) has to be adjusted, the augmented error $\varepsilon_1(t)$ must contain an additional gain as shown in Fig. 2.2(b).

The augmented error can now be expressed as

$$\varepsilon_1 = \frac{k_p}{k_m} \phi^T \zeta + \psi_1 e_2 + \delta_2(t), \quad \delta_2(t) = \frac{k_p}{k_m} \left(\theta^{*T} \zeta - W_m(s) \theta^{*T} \omega \right)$$
 (2.11)

where $\xi = W_m(s) I \omega$, $k_1(t) = k_p/k_m + \phi_1(t)$ and $\delta_2(t)$ is an exponentially decaying term due to initial conditions. The adaptive law for adjusting $\phi(t)$ and $\phi(t)$ are given by

$$\dot{\phi} = -sgn(k_p) \frac{\varepsilon_1 \xi}{1 + \overline{\xi}^T \overline{\xi}}$$

$$\dot{\psi}_1 = -\frac{\varepsilon_1 e_2}{1 + \overline{\xi}^T \overline{\xi}}$$
(2.12)

where $\overline{\xi} = W_m(s)I\overline{\omega}$. These adaptive laws assure the global boundedness of all the signals in the overall adaptive system and that $\lim_{t\to\infty}e_1(t)=0$.^[3]

III. MODEL REFERENCE ADAPTIVE CONTROL WITH FUZZY COMPENSATOR

3.1 A Structure of Model Reference Adaptive Controller with Fuzzy Compensator(MRACF)

A structure of MRACF is shown in Fig. 3.1. The control input $u_p(t)$ can be obtained by adding $u_p(t)$ generated by fuzzy compensator to u(t) generated by MRAC. But it should

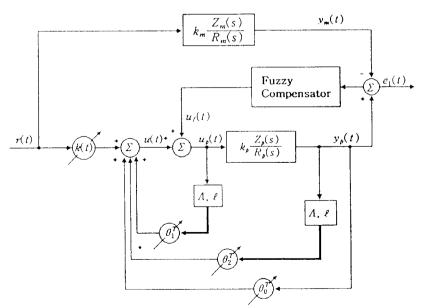


Fig. 3.1 A structure of MRACF suggested in this paper



3.2 Design of Fuzzy Compensator^[4]

The configuration of the fuzzy compensator suggested is shown in Fig. 3.2. With three inputs of fuzzy compensator $e_1(nT)$, $r_1(nT)$, and $a_1(nT)$, the structure of the fuzzy compensator can be composed of two independent parallel fuzzy control blocks which contain fuzzy control rules and defuzzifier. The incremental output of the fuzzy compensator is formed by algebraically adding the two outputs of fuzzy control blocks.

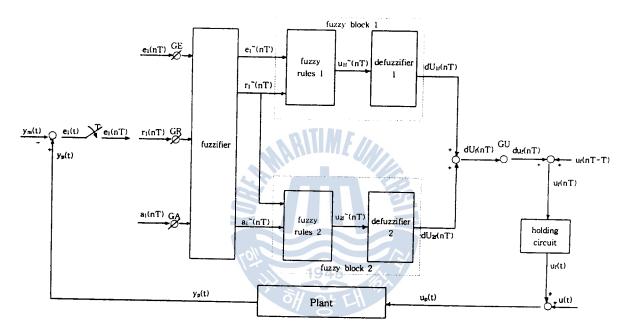


Fig. 3.2 Configuration of the fuzzy compensator

The notations employed are as followings:

$$\begin{split} e_1(nT) &= sampling \; [\; e_1(t)\;] \; \mid_{t=nT}, \; e_1^* = GE*e_1(nT) \\ r_1(nT) &= \; [e_1(nT) - e_1(nT - T)\;] \; /T, \; r_1^* = GR*r_1(nT) \\ a_1(nT) &= \; [r_1(nT) - r_1(nT - T)\;] \; /T \\ &= \; [e_1(nT) - 2e_1(nT - T) + e_1(nT - 2T)\;] \; /T^2 \\ a_1^* &= \; GA*a_1(nT)\;, \; u_f(nT) = du_f(nT) + u_f(nT - T) \\ du_f(nT) &= \; GU_f*dU_f(nT)\;, \; dU_f(nT) = dU_1f(nT) + dU_2f(nT) \end{split}$$



3.2.1 Fuzzification algorithm for scaled inputs of fuzzy compensator

The fuzzification algorithm for scaled inputs is shown in Fig. 3.3(a). The fuzzy set "error" has two members EP(error_positive) and EN(error_negative); the fuzzy set "rate" has two members RP(rate_positive) and RN(rate_negative); the fuzzy set "acc" also has two members AP(acc_positive) and AN(acc_negative).

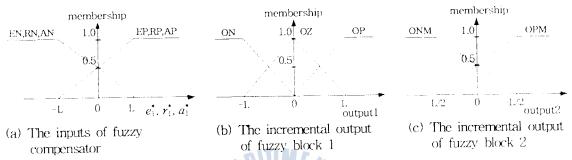


Fig. 3.3 Fuzzification algorithm for fuzzy compensator

The fuzzy set "output1" has three members OP(output_positive), OZ(output_zero) and ON(output_negative) shown as in Fig. 3.3(b) for the fuzzification of incremental output of fuzzy block 1. The fuzzy set "output2" has two members OPM(output_positive_middle) and ONM(output_negative_middle) as shown in Fig. 3.3(c) for the fuzzification of incremental output of fuzzy block 2.

3.2.2 Fuzzy rules and fuzzy logics for evaluation of the fuzzy rules

For fuzzy block 1, four linear fuzzy rules are given as:

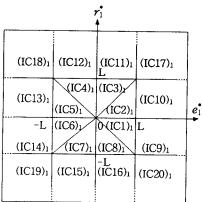
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(R1)_1: if error = EP and rate = RP then output = ON (R2)_1: if error = EP and rate = RN then output = OZ (R3)_1: if error = EN and rate = RP then output = OZ (R4)_1: if error = EN and rate = RN then output = OP
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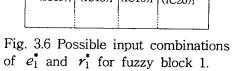
For fuzzy block 2, four linear fuzzy rules are given as:

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(R1)_2: if rate = RP and acc = AP then output = ONM (R2)_2: if rate = RP and acc = AN then output = OPM (R3)_2: if rate = RN and acc = AP then output = ONM (R4)_2: if rate = RN and acc = AN then output = OPM
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The eight different combinations of scaled error and scaled rate constituting inputs to the rules are shown graphically in Fig. 3.6 for the block 1. For the block 2, the eight different combinations of scaled rate and scaled acc are shown in Fig. 3.7.







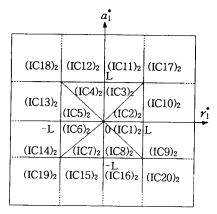


Fig. 3.7 Possible input combinations of r_1^* and a_1^* for fuzzy block 2.

3.2.3 Defuzzification algorithm

Thus the defuzzified output of a fuzzy set is defined as

$$dU = \frac{\sum (\text{membership of member}) * (\text{value of member})}{\sum (\text{memberships})}$$
(3.1)

The incremental output of the fuzzy block 1 at sampling time nT, $dU_{1f}(nT)$, can be described by the following two equations.

$$If GR * |r_1(nT)| \le GE * |e_1(nT)| \le L,$$

$$dU_{1f}(nT) = -\frac{0.5 * L}{2L - GE * |e_1(nT)|} [GE * e_1(nT) + GR * r_1(nT)]$$
(3.2)

$$If GE * |e_1(nT)| \le GR * |r_1(nT)| \le L,$$

$$dU_{1}(nT) = -\frac{0.5 * L}{2L - GR * |r_1(nT)|} [GE * e_1(nT) + GR * r_1(nT)]$$
(3.3)

If scaled error and/or scaled rate are not within the interval [-L, L] of the fuzzification algorithm shown in Fig. 3.3, the incremental output of the fuzzy block 1 is as listed in Table 3.3.

In a similar way, when the defuzzification algorithm is applied to Table 3.2, the incremental output of the fuzzy block 2 at sampling time nT, $dU_{2f}(nT)$, can be given by the following two equations.

$$If GA * |a_1(nT)| \le GR * |r_1(nT)| \le L ,$$

$$dU_{2f}(nT) = -\frac{0.25 * L}{2L - GR * |r_1(nT)|} [GA * a_1(nT)]$$
(3.4)



If
$$GR * |r_1(nT)| \le GA * |a_1(nT)| \le L$$
,

$$dU_{2f}(nT) = -\frac{0.25 * L}{2L - GA * |a_1(nT)|} [GA * a_1(nT)]$$
(3.5)

If scaled rate and/or scaled acc are not within the interval [-L, L] of the fuzzification algorithm, the incremental output of the fuzzy block 2 is as listed in Table 3.4.

Table 3.3 The incremental output of the fuzzy compensator when e_1^* and/or r_1^* are not within the interval [-L, L] of the fuzzification algorithm.

, are not	
Input combinations as shown in Fig. 3.6	Incremental output of the fuzzy block 1, $dU_{1f}(nT)$
as snown in Fig. 5.0	
$(IC9)_1$, $(IC10)_1$	$-[GR * r_i(nT) + L]/2$
(IC11) ₁ , (IC12) ₁	-[GE * $e_1(nT) + L]/2$
$(IC13)_1$, $(IC14)_1$	$-[GR * r_1(nT) - L]/2$
$(IC15)_1$, $(IC16)_1$	$-[GE * e_1(nT) - L]/2$
(IC17) ₁	t
$(IC18)_1$, $(IC20)_1$	0
(IC19) ₁	I Co L

Table 3.4 The incremental output of the fuzzy controller when r_1^* and/or a_1^* are not within the interval [-L, L] of the fuzzification algorithm.

Input combinations as shown in Fig. 3.7	Incremental output of the fuzzy block 2, dU ₂ (nT) -0.5 * GA * a ₁ (nT)
(IC1) ₂ , (IC10) ₂ , (IC13) ₂ , (IC14) ₂ (IC11) ₂ , (IC12) ₂ , (IC17) ₂ , (IC18) ₂	-0.5 * L
$(IC15)_2$, $(IC16)_2$, $(IC19)_2$, $(IC20)_2$	0.5 * L

Conclusively, the incremental output of fuzzy compensator can be divided into four different forms according to the following conditions:

1) If
$$GR * |r_1(nT)| \le GE * |e_1(nT)| \le L$$
 and $GA * |a_1(nT)| \le GR * |r_1(nT)| \le L$,

$$du_{f}(nT) = -\frac{0.5*L*GU}{2L - GE*|e_{1}(nT)|} [GE*e_{1}(nT) + GR*r_{1}(nT)]$$

$$-\frac{0.25*L*GU}{2L - GR*|r_{1}(nT)|} [GA*a_{1}(nT)]$$
(3.6)

2) If
$$GR * |r_1(nT)| \le GE * |e_1(nT)| \le L$$
 and $GR * |r_1(nT)| \le GA * |a_1(nT)| \le L$,



$$du_{f}(nT) = -\frac{0.5*L*GU}{2L - GE*|e_{1}(nT)|} [GE*e_{1}(nT) + GR*r_{1}(nT)]$$

$$-\frac{0.25*L*GU}{2L - GA*|e_{1}(nT)|} [GA*e_{1}(nT)]$$
(3.7)

3) If
$$GE * |e_1(nT)| \le GR * |r_1(nT)| \le L$$
 and $GA * |a_1(nT)| \le GR * |r_1(nT)| \le L$,

$$du_{f}(nT) = -\frac{0.5*L*GU}{2L - GR*|r_{1}(nT)|} [GE*e_{1}(nT) + GR*r_{1}(nT)]$$

$$-\frac{0.25*L*GU}{2L - GR*|r_{1}(nT)|} [GA*a_{1}(nT)]$$
(3.8)

4) If
$$GE * |e_1(nT)| \le GR * |r_1(nT)| \le L$$
 and $GR * |r_1(nT)| \le GA * |a_1(nT)| \le L$,

$$du_{f}(nT) = -\frac{0.5*L*GU}{2L - GR*|r_{1}(nT)|} [GE*e_{1}(nT) + GR*r_{1}(nT)]$$

$$-\frac{0.25*L*GU}{2L - GA*|a_{1}(nT)|} [GA*a_{1}(nT)]$$
(3.9)

If scaled error, rate and/or acc are not within the interval [-L, L] the incremental output of the fuzzy compensator is obtained from the combinations of incremental outputs for the fuzzy blocks given in Table 3.3 and Table 3.4.

Thus far, a fuzzy compensator which supervises standard MRAC scheme was established. Although the derivation process is based on the design process of general fuzzy logic compensator, the resultant incremental output of fuzzy compensator, $dU_f(nT)$, has analytical forms with time-varying gains rather than linguistic forms.

Therefore, it is convenient to apply only if input and output scalar GE, GR, GA, and GU are selected appropriately, while the performance supervising standard MARC scheme against nonlinearities and uncertainties may be superior relatively because its structure is time-varying nonlinear PID type. [4]

IV. SIMULATIONS AND RESULTS

When the plant is nonlinear and time-varying in the presence of bounded disturbance, the adaptive control was simulated using MRAC, MRRC, and MRACF scheme.



plant: $y = -2y + (3.0 + 0.5 * \cos t)y^2 + u + u + v$

model: $\dot{y} = -y + r$

input: $r(t) = 5\cos t + 20\cos 5t$

disturbance: $v(t) = 0.5 \sin t + e_1 \cos 2t + 0.5 e_1^2 \cos t$

plant: $y = y + (2.0 + \cos t)y^2 + u + u + v$

model: y = -y + r

input: $r(t) = 5\cos t + 20\cos 5t$

disturbance: $v(t) = 0.5\sin t + e_1\cos 2t + 0.5e_1^2\cos t$

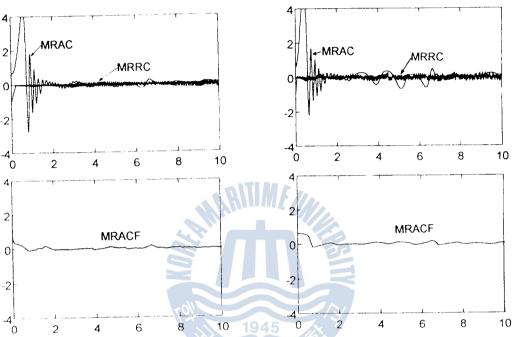


Fig. 4.1 Comparison of the performances when Fig. 4.2 Comparison of the performances when the plant is stable nonlinear time-varying the plant is unstable nonlinear time-varying

In the standard MRAC, the output error e_1 does not converge to zero and oscillates as $t\rightarrow\infty$. In the MRRC, the output error converges fastly but not to zero by chattering accompanied, so it exhibits poor steady state response. The MRACF have a better transient response and steady state response than those of MRAC and MRRC.

For MRACF scheme, it is shown that the output error converge to zero and the parameter error is bounded with persistent excitation. Since the input generated by fuzzy compensator is added to the input of the adaptive controller, parameter error does not converge to zero even when input is persistently excited. Nevertheless, the transient response and steady-state response are good in spite of nonlinear time-varying and nonminimum phase plant with disturbance.

V. CONCLUSION

In this paper, a MRACF scheme was suggested in order to improve performance and assure stability of overall system for nonlinear and time-varying systems with unknown



disturbance. The scheme improved performance by using fuzzy compensator supervising the MRAC and assured global stability in the sense that the output error and all internal signals were bounded by fuzzy compensator, while fuzzy compensator should be designed stably.

In simulation study, the MRRC scheme exhibits a fast rising time, while steady-state response is not good as shown in Fig. 4.1-4.2. MRACF scheme exhibits better performance than those of other adaptive schemes as shown in Fig. 4.1-4.2.

The scheme can be easily applied because the requirement of prior information is only equal to that of the standard MRAC. And fuzzy compensator designed in this paper is analytical and PID type, the MRACF scheme has simplicity in applications to nonlinear time-varying real systems and assures performance improvement.

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