Journal of the Industrial Technology Research Institute.
Korea Maritime University. Vol. 15. 1997.

A Sliding Mode Controller with Neural Network and Fuzzy Boundary Layer

Min – ho Lee *

Department of Electrical Engineering, Korea Maritime University
1 Dongsam – Dong Yeongdo – Gu Pusan 606 – 791, Korea
E-mail : mhlee@hanara.kmaritime.ac.kr

Abstract

A sliding mode controller with a neural network and a fuzzy boundary layer is proposed. A multilayer neural network is used for constructing the inverse identifier which is an observer of the uncertainties of a system. Also, fuzzy boundary layer is introduced to make the continuous control input of sliding mode controller combined with the neural inverse identifier. The proposed control scheme not only reduces an effort for finding an unknown dynamics of a system but also alleviates the chattering problems of the control input. Computer simulation reveals that the proposed approach is effective to alleviate the chattering problem of the control input.

1. Introduction

The sliding mode controller is a powerful nonlinear controller[1, 2].However, this controller has several important drawbacks such as high control authority and control chattering. Since the chattering involves extremely high control activity and may excite high frequency dynamics neglected in the modeling, many approaches have been reported to alleviate the chattering phenomenon[3, 4]. The so – called boundary layer approach, which generates the continuous control law, is one example[3]. When the magnitude of the uncertainty bound of a system is large, the chattering magnitude of control input is also large. Thus, the width of the boundary layer should be large to make the continuous control input, and resultantly it makes

* Department of Electrical Engineering, Korea Maritime University

a large steady state error. Therefore, it is necessary to have trade－off between the steady－state error and the chattering magnitude. Another approach is the adaptive chattering alleviation control algorithm[4]. This approach has difficulty to apply to fast time－varying systems.

In this paper, a new sliding mode controller using a neural network and a fuzzy logic is proposed. To improve the chattering problem, it requires more information of unknown dynamics. Since the neural network can approximate nonlinear functions in L^p and has good generalization capabilities[5, 6], it can be used for construction of an inverse identifier which generates the control input to reduce the chattering magnitude. A multilayer neural network with the error back propagation learning algorithm is used for construction of the neural inverse identifier, and operates as an observer of an uncertain information. It gives not only the small chattering magnitude of control input but also convenience for design of the control input, because the control input for chattering depends on the maximum magnitude of inverse identification error instead of a priori knowledge of the bounds in external disturbances and parameter variations. A fuzzy boundary layer of which the width is determined by the learning accuracy of the neural inverse identifier, is introduced to generate the continuous control input of the proposed sliding mode controller with a neural network.

2. A sliding mode controller with neural network

The system with sliding mode control is described as follows :

$$\dot{x}_i = x_{i+1} \qquad i=1, 2, \cdots, n-1 \tag{1}$$

$$\dot{x}_n = -\sum_{i=1}^{n} a_i x_i + bu - f \tag{2}$$

$$= -\sum_{i=1}^{n} a_i^0 x_i + bb^0 u - G \tag{3}$$

where

$$G = -\sum_{i=1}^{n} \Delta a_i x_i + \Delta b u + f \quad \text{and} \quad \dot{x} = r - x_1 \tag{4}$$

with

$$z(0) = [x_1(0) + \frac{c_2}{c_1} x_2(0) + \cdots + + \frac{c_n}{c_1} x_n(0)]/k_1 \tag{5}$$

where x_i is the state or the output of a system, a_i and b represent the i－th plant parameter, a_i^0 and b^0 are the known nominal values of a_i and b, respectively. Δa_i and Δb are the deviations from the known nominal values and r represents the reference input signal. G represents the unknown information of the system, and $\dot{z}$ is the integral augmented state with suitable initial condition for solving the reaching phase problem in the sliding mode controller[1]. The c_i is a constant coefficient of the sliding line which will be determined by a method. The u is a piecewise linear control function of the following form

$$u = \begin{cases} u^{+}(x, t) & \text{if} \quad \sigma > 0 \\ u^{-}(x, t) & \text{if} \quad \sigma < 0 \end{cases} \tag{6}$$

where σ is the switching function given by

$$\sigma = c_1(x_1 - k_1 z) + \sum_{i=2}^{n} c_i x_i \quad \text{and} \quad c_n = 1 \tag{7}$$

Let the control input u be

$$u = u_{eq} + \Delta u \tag{8}$$

where u_{eq}, called the equivalent control input, is defined as the solution of the equation $\dot{\sigma} = 0$ under the conditions that $f = 0$, $a_i = a_i^0$, and $b = b^0$, that is,

$$u_{eq} = [c_1 k_1 (r - x_1) - \sum_{i=2}^{n} c_{i-1} x_i]/b^0 + \sum_{i=1}^{n} a_i^0 x_i / b^0 \tag{9}$$

The control function Δu is used to eliminate the presences of Δa_i, Δa b, and f so as to guarantee the existence of the sliding mode. This function is considered as

$$\Delta u = \sum_{i=1}^{n} \Psi_i x_i + \Phi \tag{10}$$

where

$$\Psi_i = \begin{cases} \alpha i & \text{if} \quad \sigma x_i > 0 \\ \beta_i & \text{if} \quad \sigma x_i < 0 \end{cases}$$

and

$$\Phi = \begin{cases} \gamma & \text{if} \quad \sigma > 0 \\ \delta & \text{if} \quad \sigma < 0 \end{cases}$$

From the existence and reachability of the sliding motion as shown in equation (11)[11]

$$\sigma \dot{\sigma} < 0, \tag{11}$$

the following equation is obtained.

$$\sigma \dot{\sigma} = \sigma\, [c_1(x_2 - k_1(r - x_1)) + \sum_{i=2}^{n-1} c_i x_{i+1} + \{ - \sum_{i=1}^{n} a_i x_i + b(\frac{1}{b_0} [c_1 k_1 (r - x_1)$$

$$- \sum_{i=2}^{n} c_{i-1} x_i] + \frac{1}{b^0} \sum_{i=1}^{n} a_i^0 x_i + \sum_{i=1}^{n} \Psi_i x_i \quad + \Phi)\} - f] < 0 \tag{12}$$

Thus, the conditions for satisfying the inequality (11) are

$$\Psi_i=\begin{cases}\alpha_i<\Delta b(c'_i-a_i)/(bb^0)+\Delta a_i/b & \text{if } \sigma x_i>0\\ \beta_i>\Delta b(c'_i-a_i)/(bb^0)+\Delta a_i/b & \text{if } \sigma x_i>0\end{cases}$$

where

$$c'_i=\{c_1k_1,c_1,\cdots,c_n\} \quad i=1,2,\cdots,n \tag{13}$$

and

$$\Psi=\begin{cases}\gamma<f/b-\Delta bc_1k_1r/(bb^0) & \text{if } \sigma>0\\ \delta<f/b-\Delta bc_1k_1r/(bb^0) & \text{if } \sigma>0\end{cases}$$

As shown in the above equations, the chattering magnitude of the control input is affected by the unknown dynamics such as the parameter uncertainties and the external disturbances. Therefore, if the uncertainty terms are inverse identified, it is expected that the chattering magnitude can be alleviated by using an additional control input generated by the inverse identified model[7]. We use the higher order multilayer neural network with the error back propagation learning algorithm for construction of the inverse identifier[7]. The error function is defined by the square of difference between the external input and the sum of the outputs which are obtained from the nominal dynamics and the neural network[8,9].

Assume that the bound of variation of the training input δu has a sufficiently large value to make u^* contained in the region Ω of the training input $u_0+\delta u$, where u^* is the exact control input for a desired trajectory and u_0 is the input obtained from the nominal dynamics for the desired trajectory. Then, if the neural network for inverse identification is trained with the following accuracy $||u-(u_{norm}+u_n)||_\infty<\varepsilon$, where the u_{norm} and the u_n represent the outputs of nominal dynamics and neural network, respectively. The following equations present that the unknown dynamics of the system can be compensated by the additional control input which is made by neural inverse identifier[8]. From the equations (1),(2) and (3),

$$u=\{\dot{x}_n+\sum_{i=1}^{n} a_ix_i+f\}/b \tag{14}$$

and

$$u_{norm}=\{\dot{x}_n+\sum_{i=1}^{n} a_i^0x_i\}/b^0 \quad \text{and} \quad u_n=N(w) \tag{15}$$

Then,

$$||u-(u_{norm}+u_n)||_\infty$$

$$=||\{\dot{x}_n+\sum_{i=1}^{n}a_ix_i+f\}/b-\{\dot{x}_n+\sum_{i=1}^{n}a_i^0x_i\}/b^0-N(w)||_\infty \tag{16}$$

$$=||(\frac{\dot{x}_n+\sum_{i+1}^{n} a_i^0 x_i}{b^0}+\frac{b^0}{b^0+\Delta_b}-1)+\frac{G-\Delta bu}{b^0+\Delta b}-N(w)||_\infty \tag{17}$$

$$=||(\frac{\dot{x}_n+\sum_{i+1}^{n} a_i^0 x_i}{b^0}+\frac{b^0}{b^0+\Delta_b}-1)+\frac{G-\Delta bu}{b^0+\Delta b}-N(w)||_\infty \tag{18}$$

$$=||(\frac{-\Delta b}{b^0+\Delta_b})(u+u_{norm})+\frac{G}{b^0+\Delta_b}-N(w)||_\infty<\varepsilon \tag{19}$$

If Δb is zero, then the uncertainty of system, G, is represented by the neural network, $N(w)$, as $||G/b^0-N(w)||_\infty<\varepsilon$. Since the control input chattering is alleviated from the magnitude of a priori knowledge of uncertainty to the maximum training error of the neural inverse identifier, it is important to use higher order multilayer neural network for accurate inverse modeling.

Let the control input $u=u_{eq}+u_n+\Delta u$, where Δu is the chattering input for compensation of training error ε and the uncertainty Δb which directly affects the control input. From the existence and reaching condition of sliding mode and the integral augmented sliding line as shown in equation (7) and (11), respectively,

$$\sigma\dot{\sigma}=\sigma[c_1(x_2-k_1(r-x_1))+\sum_{i=2}^{n-1}c_i x_{i+1}+\{-\sum_{i=1}^{n}a_i x_i+b(\frac{1}{b_0}[c_1k_1(r-x_1)\sum_{i=2}^{n}c_{i-1}x_i]$$
$$+\frac{1}{b^0}\sum_{i=1}^{n}a_i^0 x_i+u_n+\Delta_u)\backslash\}-f] \tag{20}$$

From the equation (19), u_n can be represented as shown in equation (21).

$$u_n=u-u_{norm}-\varepsilon'$$
$$=\frac{\dot{x}_n+\sum_{i+1}^{n}a_i x_i+f}{b}-\frac{\dot{x}_n+\sum_{i+1}^{n}a_i^0 x_i}{b^0}-\varepsilon' \tag{21}$$

where $||\varepsilon'||_\infty<\varepsilon$. Then, the derivative of the sliding line is

$$\dot{\sigma}=[c_1(x_2-k_1(r-x_1))+\sum_{i=2}^{n-1}c_i x_{i+1}+\sum_{i=2}^{n}a_i x_i+b(\frac{1}{b_0}[c_1k_1(r-x_1)-\sum_{i=2}^{n}c_{i-1}x_i]+\frac{1}{b_0}\sum_{i=1}^{n}a_i^0 x_i$$
$$+\frac{\dot{x}_n+\sum_{i=1}^{n}a_i x_i+f}{b}-\frac{\dot{x}_n+\sum_{i=1}^{n}a_i^0 x_i+f}{b^0}-\varepsilon'+\Delta_u)\}-f] \tag{22}$$

$$=-(\Delta b/b^0)\sum_{i=2}^{n}c_{i-1}x_i+\dot{x}_n+c_1k_1x_1+(\Delta b)/(b^0)c_1k_1r-b\varepsilon'+b\Delta u \tag{23}$$

$$=-(\Delta b/b^0)\sum_{i=1}^{n+1}c_i' x_i\}+(\Delta b)/(b^0)c_1k_1r-b\varepsilon'+b\backslash\Delta_u \tag{24}$$

where

$$c_i'=\{c_1k_1, c_1, c_2, \cdots, c_{n-1}, 1\} \qquad i=1, 2, \cdots, n+1 \tag{25}$$

Let be the chattering control input $\Delta u=\sum_{i=1}^{n+1}\Psi_i x_i+\Phi$ which is used for both canceling the error of inverse identification and the effect of the uncertainty Δb, where Ψ_i and Φ are given in the form of equation (10). Then, the equation (24) becomes

$$\dot{\sigma} = \sum_{i=1}^{n+1} \{ b\Psi_i - \frac{\Delta b}{b^0} c' \} x_i + \frac{\Delta b}{b^0} c_1 k_1 r - b\varepsilon' + b\Phi. \qquad (26)$$

If we assume that the bound of Δb is known, %and the system state $\dot{x}_n$ is measurable, the following equations are obtained from the existence and reachability condition of sliding mode in equation (11),

$$\Psi_i = \begin{cases} \alpha_i < \dfrac{\Delta b}{b^0(b^0+\Delta b)} c'_i & \text{if} \quad \sigma x_i > 0 \\ \beta_i > \dfrac{\Delta b}{b^0(b^0+\Delta b)} c'_i & \text{if} \quad \sigma x_i < 0 \end{cases}$$

and

$$\Phi_i = \begin{cases} \gamma < \dfrac{1}{b^0(b^0+\Delta b)} \{c_i k_1 r - b\varepsilon' & \text{if} \quad \sigma > 0 \\ \delta > \dfrac{1}{b^0(b^0+\Delta b)} \{c_i k_1 r - b\varepsilon' & \text{if} \quad \sigma < 0 \end{cases}$$

If Δb is zero, since $|| G/b^0 - N(w) || < \varepsilon'$ from the equation (19), one obtains

$$\Delta u = \gamma < \varepsilon' \qquad \text{if} \quad \sigma b^0 > 0 \qquad (27)$$

$$\Delta u = \delta > \varepsilon' \qquad \text{if} \quad \sigma b^0 < 0 \qquad (28)$$

Then, we design the optimal sliding line by minimizing the quadratic performance index function for satisfying the minimum error and minimum energy[10]. It is clear that the problem of designing a system with desirable properties in the sliding mode can be regarded as a linear state feedback design problem[10]. Thus, an optimal gain matrix of the sliding line is obtained by the optimal linear regulator technique using Riccati equation.

In order to get the continuous control input, a fuzzy boundary layer technique{Palm} is incorporated around at the optimal sliding line with the width of ε'' which is maximum inverse identification error of the higher order multilayer neural network. If Δb is zero, ε'' is equal to the ε'. Using the higher order neural inverse identifier and fuzzy boundary layer technique, one can obtain the effective sliding mode controller which does not needs a complete knowledge of the bounds in system uncertainty and also has smaller steady state error.

3. Simulation

Let us consider the following simple system

$$\dot{X}_i = AX + BU - f \tag{29}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta ai = \pm 0.5,\ \Delta b = 0, \quad \text{and} \quad f = \sin(x_1). \tag{30}$$

The performance index function for the optimal sliding line is

$$J = \int_0^{\infty} (5X_1^2 + X_2^2 + 4u^2)dt \tag{31}$$

From the Riccati equation, the optimal gain for the sliding line C is [– 0.29125, – 0.13575] and sampling time is 5 *msec*. Fig. 1 shows the control results of the regulation problem. Solid line represents the reference step input, and long dashed line and dashed line show the controlled output with and without the neural inverse identifier, respectively. The dotted line shows the controlled output when fuzzy boundary layer is used for making the continuous control inputs. Fig. 2 and 3 show the control input without and with the neural inverse identifier, respectively. As shown in Fig. 2 and 3, the chattering magnitude of the control input successfully alleviated without much degradation of control performance. Fig. 4 shows the continuous control input with neural inverse identifier and fuzzy boundary layer. As shown in Fig. 4, the fuzzy boundary layer which has narrower width by the action of neural inverse

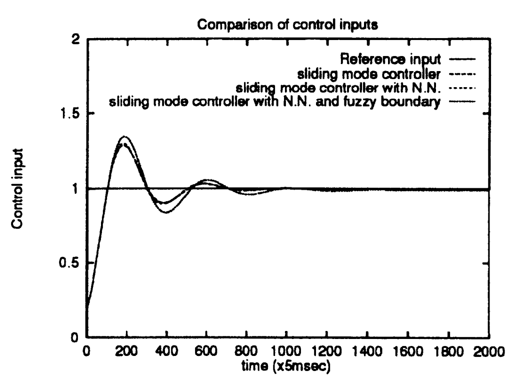

Fig. 1. Control results of variable structure controller.

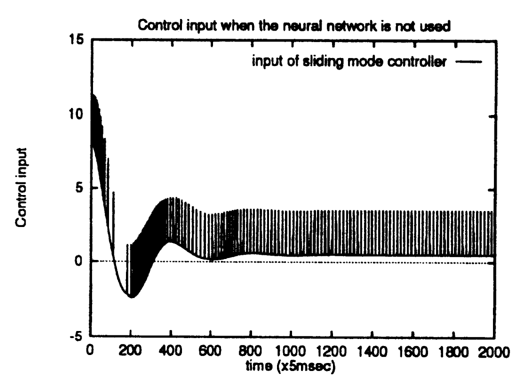

Fig. 2. Control input of the sliding mode controller without neural network

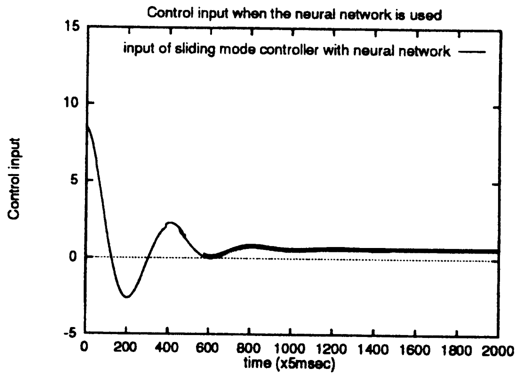

Fig. 3. Control input of the sliding mode controller with neural network.

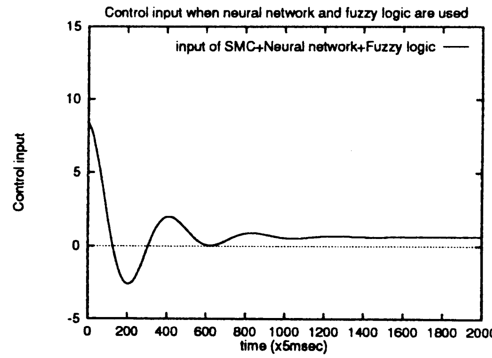

Fig. 4. Control input of the sliding mode controller with neural network and fuzzy boundary layer.

identifier generates the effective continuous control input.

4. Discussion

A new approach for chattering alleviation is proposed and demonstrated by computer simulation. We successfully alleviated the chattering magnitude of the control input of the sliding mode controller by neural inverse identifier. Moreover, this approach does not require a priori knowledge of the bounds of parameter variations and external disturbances. Using the fuzzy boundary layer technique, an effective continuous control input can be obtained by incorporating the sliding mode controller with the neural inverse identifier.

However, sufficient data for training of the neural network is necessary and the method for obtaining the data must be considered. Also, if a system does not have inverse, the training of neural inverse identifier with error back propagation algorithm is impossible and another neural network with new training scheme is necessary and under investigation.

References

[1] U. Itkis. *Control systems of variable structure*, John Wiley & Sons, INC., New York ,1976.

[2] J.Y. Hung, W. Gao, and J.C. Hung. Variable structure control : a survey, *IEEE Trans. on Industrial Electronics*, 40(1), 1993.

[3] J.J. Slotine. Sliding controller design for nonlinear systems, *International Journal of Control*, 40(2):421 – 434, 1984.

[4] F.J. Chang, and S.H. Twu. Adaptive chattering alleviation of variable structure systems control, *IEE Proceedings Part D*, 137(1):31 – 39, 1990.

[5] K. Hornik. Approximation capabilities of multi – layer feedforward networks, *Neural Networks*,

4(2):251 – 257, 1991.

[6] E.B. Baum and D. Hausser. What size net gives valid generalization ?, *Neural Information Processing Systems*, 1:81 – 90, 1989.

[7] M. Lee, B.J. Choi, S.Y. Lee and C.H. Park, and B.K. Kim. Variable structure controller with higher order multilayer neural inverse identifier, *International Conference on Artificial Neural Networks*, 2:1207 – 1210, 1994.

[8] M. Lee, S.Y. Lee, C.H. Park. Neural controller of nonlinear dynamic systems using higher order neural networks, *Electronics Letters*, 28(3):276 – 277, 1992.

[9] M. Lee, S.Y. Lee, C.H. Park. Higher – order neuro – controller of nonlinear dynamic systems, *1992 Annual Conf. of Japanese Neural Network Society*,, 205 – 206, 1992.

[10] T.L. Chern and Y.C. Wu. An optimal variable structure control with integrated compensation for electrohydraulic position servo control system, *IEEE Trans. on Industrial Electronics*, 39(5):460 – 464, 1992.

[11] R. Palm. Sliding mode fuzzy control, *IEEE Int. Con. on Fuzzy Systems*, 519 – 526, 1992.